Physics

part 2

Grade 11

SPRIX

Physics

Egyptian Baccalaureate
2nd Year

The International Baccalaureate™
has reviewed the pedagogical and assessment approaches underpinning this textbook.

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Ministry of Education and Technical Education
New Administrative Capital
Cairo, Egypt

Introduction

Physics is the search for the smallest number of ideas that explain the largest number of things. A bicycle pump that grows warm in your hand, a rubbed balloon that lifts scraps of paper, a compass needle that jumps the moment a switch closes, the spark that fires an engine — all of them yield to a handful of principles. Putting those principles within your reach is the work of this book.

This Grade 11 Physics textbook has been developed in alignment with the vision of the Egyptian Baccalaureate System and in accordance with the International Baccalaureate framework. It is designed to build not only what you know, but how you come to know it: to observe carefully, to predict before you are told, to measure with your own hands, and to name an idea only after you have experienced it.

This part you will travel through three connected landscapes. In **Gases and Heat** you will follow a gas from the pressure and temperature you can read off a gauge down to the molecules whose collisions produce them. You will assemble the ideal gas law from measurements you take yourself, account for every joule that enters or leaves a trapped gas, and learn to state exactly what is conserved when heat and work are exchanged.

In **Electricity and Magnetism** you will begin with charge at rest: the force between two charges, the field that carries that force, the potential that measures it, and the capacitor that stores it. You will then set charge moving, solve networks of currents with Kirchhoff's rules, and discover that a current and a magnet are two views of one thing — a current deflects a compass, a magnet pushes a current-carrying wire, and a field that changes drives a current of its own.

In **The Quantum Nature of Light and Matter** you will meet the edge of the classical picture. Light will arrive in countable packets that knock electrons out of a metal, X-rays will be produced and then diffracted by the orderly planes of a crystal, and matter itself will show the interference that belongs to waves.

Each lesson follows the same deliberate sequence. It opens with a question worth answering and a phenomenon worth explaining, and asks you to commit a prediction in writing before you read on. It then hands you the apparatus: you explore, collect your own data, and look for the pattern in it. Only then is the model named, stated precisely, and put to work in examples you can follow line by line. Practice follows in graded steps — a first attempt, then a new context, then an exam-style question — so that understanding is built rather than memorized.

Two habits run through every unit. The first is the **Nature of Science**: short accounts of how a result was actually won — who measured it, what they got wrong, and how the correction came. The second is the discipline of saying where a model stops. The ideal gas law describes a real gas only while it stays dilute; Coulomb's law holds only while the bodies are far apart compared with their size; a coil behaves as a fixed inductance only until its iron core saturates. Knowing the limits of an equation is part of knowing the equation.

You will also find this physics in the devices around you. The same self-induction that snaps a spark across an opening switch fires the plug in a car engine and protects a computer from a surge on the line; the same force on a current-carrying wire moves the cone of a loudspeaker and grips steel in a scrap-yard crane. The intention is simple: the laws you are studying are not confined to a laboratory.

Mathematics is used here as physicists use it — as a language for reasoning, not a list of formulas to recall. Symbols are defined before they are used, units are carried through every calculation, and each worked example closes by asking whether the answer is reasonable.

Approach this book with a pencil in your hand. Write the prediction even when you are unsure of it: a wrong prediction that you examine will teach you more than a correct answer you were handed. Take your own measurements seriously enough to ask why they scatter. Physics rewards that kind of attention, and it is well within your reach.

How to use this book

Every lesson is built from the same parts, in the same order. Learn them once, and you can read any lesson in the book. The next page shows the learning loop those parts run on.

1 **Lesson question**
The everyday puzzle the whole lesson sets out to answer — and returns to answer in full at the end.

2 **You will learn to**
The skills you will be able to perform by the end.

3 **The phenomenon — predict first**
A real scene; you commit a written prediction before reading on.

4 **Explore**
A short hands-on activity where your own data becomes the evidence.

5 **From observation to model**
Where the physics is built: the reasoning that turns what you saw into a law.

6 **Point**
The core idea stated as a short fill-in-the-gaps summary.

7 **Equation box**
The key relations of the lesson gathered at a glance.

8 **Worked Example**
One problem solved field by field — then its twin, scaffolding faded, for you to finish.

9 **Try**
Practice problems, each with its figure beside it.

10 **In a new context**
The same idea applied to a fresh, real-world situation.

11 **Nature of Science**
How scientists came to know this idea, drawn from the original.

12 **Margin boxes**
Key idea, Working note, Key fact, Term, Note — short asides in the margin beside the text they belong to.

13 **Lesson question, answered**
The lesson's own opening question, answered in full once you have the physics to answer it.

14 **Exercise**
End-of-lesson questions to consolidate and self-check.

What makes this book different —

① **Predict before the explanation.** You write a prediction before each phenomenon, and only then read on.

② **Understand *why*.** Every worked method shows its reasoning, so you learn to think like a physicist — not just memorize the moves.

How learning works here

Read this page once, then return to it whenever you need it. It holds for every lesson in the book.

Every lesson runs on one loop

Observe → **Predict** → **Measure** → **Name** ↺

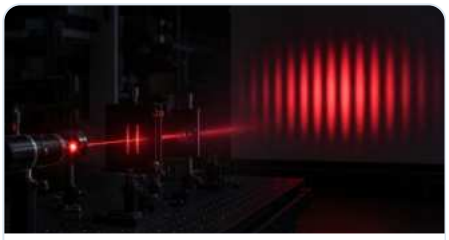
- **Observe** — a real thing happens, and it breaks what you expected.
- **Predict** — you commit a guess in writing, before you read on.
- **Measure** — you do it with your own hands and collect the numbers.
- **Name** — only now does the term arrive: earned, not memorized.

> **Commit your prediction in writing — your guesses are data, not decoration.**

Where this part sits in your physics course

C — Particulate nature of matter
this part · Gases and Heat

D — Fields
this part · Electricity and Magnetism

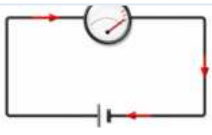


E — Nuclear and quantum physics
this part · Quantum

Your five units this part

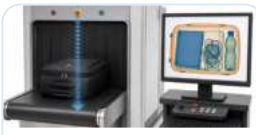
Unit 1
Gases and Heat
Lessons 1-1 → 1-6

Unit 2
Static Electricity
Lessons 2-1 → 2-6



Unit 3
Electric Current and Circuits
Lessons 3-1 → 3-2

Unit 4
Magnetism and Electromagnetic Induction
Lessons 4-1 → 4-5



Unit 5
The Quantum Nature of Light and Matter
Lessons 5-1 → 5-3

Built on part 1 — **A** Space, time and motion · **B** Wave behavior.

Tools: Tool 1 — Experimental techniques · Tool 2 — Technology · Tool 3 — Mathematics.

Inquiry cycle: Exploring and designing · Collecting and processing · Concluding and evaluating — you meet these on every Explore activity.

Academic honesty: the prediction must be your own — cite every source, and claim only your own work.

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UNIT 1

Gases and Heat

1-1	Boyle-Charles's Law	8
1-2	Equation of State for an Ideal Gas	14
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1-1 Boyle-Charles's Law

❓ **Lesson question:** How are the pressure, volume, and absolute temperature of a fixed mass of gas related?

You will learn to

1. Calculate the pressure of a gas from the force per unit area, $p = F/A$, in pascals (Pa).
2. Find the pressure of gas enclosed by a piston by applying the balance of forces on the piston.
3. Apply Boyle-Charles's law, pV/T = constant, to relate the states of a fixed mass of ideal gas before and after a change.

🔍 The phenomenon — predict before you continue

On an airplane at cruising altitude, the cabin is pressurized, but its pressure is lower than atmospheric pressure at ground level. A sealed bag of chips packed at ground level therefore swells as the trapped gas expands until a new mechanical equilibrium is reached. If the external pressure remains approximately constant, warming the gas increases its volume, whereas cooling it decreases its volume. Thus, for a fixed mass of gas, volume depends on both the pressure and the absolute temperature — the temperature measured in kelvin, T [K] = T [°C] + 273.

★ **Key idea**

Pressure p — the force acting per unit area, $p = F/A$, measured in pascals (Pa); 1 Pa = 1 N/m².

Figure 1-1-1 · A sealed bag of chips swells at cruising altitude because the regulated cabin pressure is lower than the atmospheric pressure at ground level. The trapped gas expands until a new mechanical equilibrium is reached.

Predict

(a) When a fixed mass of gas is compressed at constant temperature, does its pressure rise, fall, or stay the same?

(b) When you warm a sealed gas at constant pressure, does its volume rise, fall, or stay the same?

🔍 Explore — Sealed-air squeeze

What you'll do: trap a fixed mass of air in a syringe and change one quantity at a time to see how pressure, volume, and temperature are linked.

Time: 15 min

You will need:

- a large syringe with a cap
- a ruler or caliper
- slotted masses
- a thermometer
- a water bath

capped syringe

slotted masses

thermometer

water bath

Steps

1. With no load and the nozzle capped, read the trapped volume V on the barrel scale and the pressure p (atmospheric).
2. Add slotted masses to the plunger one at a time to raise the pressure p. Keep the temperature fixed and record V after each addition.
3. For each run work out the product $p \times V$ and enter it in the last column.
4. Return to no load, then stand the whole syringe in a water bath and warm it, reading V against the absolute temperature T at fixed pressure.

Run	Load on plunger	Pressure p (kPa)	Volume V (mL)	$p \times V$ (kPa·mL)
1	no load			
2	one mass			
3	two masses			

Warming run (step 4) — keep the load fixed so the pressure is constant, warm the trapped air in the water bath, and read the volume against the absolute temperature.

Reading	Temperature T (K)	Volume V (mL)	V/T (mL/K)
cool			
warm			
hot			

Analyze what you collected:

(a) As the pressure p rises at constant temperature, what happens to the volume V? Look down the last column — what do you notice about the product $p \times V$?

(b) In the warming run at fixed pressure, predict the shape of a graph of V against the absolute temperature T, and say what the ratio V/T should do.

✎ Write your explanation in two lines before you read the next page.

Figure 1-1-2 · A sealed syringe held vertically: a known load presses the plunger while the trapped volume V is read on the barrel scale; the nozzle is capped.

★ Working note

Measure the plunger's diameter d with the ruler once, so its area is $A = \pi d^2/4$ and each load m gives $p = p_0 + mg/A$. Let the plunger settle before each reading, keep the temperature steady while you change the load, and trust the pattern across runs more than any single measurement.

From observation to model

Pressure is force spread over an area

Pressure measures how concentrated a push is: the same force spread over a larger area presses less hard on each part of the surface, so we divide the force by the area it acts on, $p = F/A$ (Figure 1-1-3). A gas trapped under a piston does not set its pressure freely — the piston floats in equilibrium, so the gas pressure is fixed by the balance of the forces acting on that piston, worked out in the Example below.

Two proportionalities become one law

Look back at the two columns you filled. In the loading runs the product $p \times V$ barely moved while the pressure and the volume each changed a great deal; in the warming runs it was V/T that held steady. Each column is a result in its own right, but each holds only while the third quantity is frozen — and a real change moves p and T together. So make the change in two stages, and use each column only where it applies. Start at (p_1, V_1, T_1). First move the pressure to p_2 with the temperature still at T_1: the loading column gives $p_1V_1 = p_2V'$, where V' is the volume reached at that halfway stage. Now warm the gas to T_2 with the pressure held at p_2: the warming column gives $V'/T_1 = V_2/T_2$, so $V' = V_2T_1/T_2$. Substitute that into the first equation and V' — the one quantity that belonged only to the halfway stage — cancels out, leaving $p_1V_1 = p_2V_2T_1/T_2$, that is,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

The halfway stage was a scaffold; what survives it is one quantity, pV/T, that reads the same before and after. That is why a single law can stand in for two.

Point

- When a gas pushes a surface of cross-sectional area A [m²] with a force of magnitude F [N], the pressure p of the gas is expressed as **$p = \frac{F}{A}$**. The pascal (symbol Pa) is used as the unit of pressure.
- To find the pressure of a gas enclosed in a container by a piston, use the **balance of forces** acting on the piston.
- Boyle-Charles's law: for a fixed mass of gas, the volume V is inversely proportional to its pressure p and directly proportional to its absolute temperature T, so **$\frac{pV}{T}$ = constant**.
- A gas in which the size of the gas molecules and the intermolecular forces can be ignored is called an **ideal gas**. In an ideal gas, Boyle-Charles's law holds exactly.

Equation box

Pressure	$p = \frac{F}{A}$ (unit: Pa = N/m²)
Piston balance	$pA - p_0A - mg = 0 \Rightarrow p = p_0 + \frac{mg}{A}$
Boyle-Charles	$\frac{pV}{T} = \text{constant} \Rightarrow \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

★ Key fact

The pascal is defined as 1 Pa = 1 N/m². Standard atmospheric pressure is about 1.0×10^5 Pa.

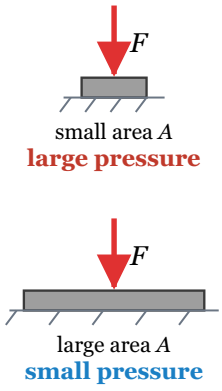

Figure 1-1-3 · Pressure is force spread over area: the same force F on a larger area A gives a smaller pressure $p = F/A$.

★ Term

Ideal gas — a useful idealization. Real gases behave almost ideally at everyday temperatures and moderate pressures; the model weakens when a gas is very cold or strongly compressed and its molecules crowd close enough for their size and their mutual forces to matter.

★ Note

Watch the units: the temperature in pV/T must be the absolute temperature in kelvin. A Celsius value gives the wrong answer, so always convert before you substitute.

🛠 Worked Example

Situation	Give each answer to 2 significant figures. (a) Air is enclosed in a cylindrical container by a smoothly moving piston with a cross-sectional area of 4.9×10^{-3} m^2 and a mass of 10 kg. Determine the pressure of the air inside the container in Pa. Take the atmospheric pressure as 1.0×10^5 Pa and the acceleration due to gravity as 9.8 m/s^2. (b) An ideal gas at 19°C, 9.6×10^5 Pa, and 3.5 L is changed to 1.2×10^5 Pa and 3.2 L. Determine its temperature in °C. (c) A gas with a volume of 0.60 m^3 and a temperature of 280 K is heated to 420 K while keeping the pressure constant. Determine the volume in m^3.
Given and required	(a) $A = 4.9\times10^{-3}$ m^2, $m = 10$ kg, $p_0 = 1.0\times10^5$ Pa, $g = 9.8$ m/s^2; find p. (b) State 1: $T_1 = 19$°C, $p_1 = 9.6\times10^5$ Pa, $V_1 = 3.5$ L; State 2: $p_2 = 1.2\times10^5$ Pa, $V_2 = 3.2$ L; find T_2 in °C. (c) $V_1 = 0.60$ m^3, $T_1 = 280$ K, $T_2 = 420$ K, constant p; find V_2.
Representation	Figure 1-1-4 shows the arrangement in (a) — air closed off by a smoothly moving piston of mass 10 kg — and Figure 1-1-5 is the balance of forces on that piston. Figure 1-1-6 shows the gas before and after the constant-pressure heating in (c); for (b), sketch the two states and label p, V, and T.
Strategy	Set up the equation of equilibrium for the forces acting on the piston to find the pressure. For a state change, use pV/T = constant before and after, and always calculate with the absolute temperature.
Annotated solution	**(a)** From the balance of forces on the piston, $pA - p_0A - mg = 0$, so $p = p_0 + mg/A = (1.0\times10^5) + (10\times9.8)/(4.9\times10^{-3})$ $= (1.0\times10^5) + (2.0\times10^4) =$ **1.2×10^5 Pa**. **(b)** Let the required temperature be T [K]. From Boyle-Charles's law, using absolute temperature (the volume units only need to match on both sides): $(9.6\times10^5)\times3.5 / (19+273) = (1.2\times10^5)\times3.2 / T$ $T = (1.2\times10^5)\times3.2\times(19+273) / [(9.6\times10^5)\times3.5] = 33.37\ldots \approx$ 33 K. Then $33 - 273 =$ **−240°C**. **(c)** Let the required volume be V [m^3] at pressure p. From Boyle-Charles's law at constant pressure, $p\times0.60 / 280 = pV / 420$, so $V = (0.60/280)\times420 =$ **0.90 m^3**.

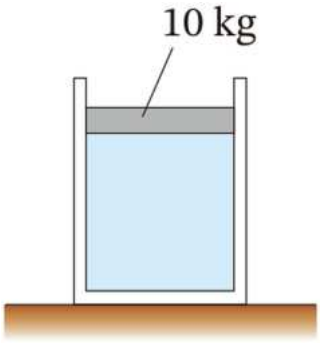

Figure 1-1-4 · Worked Example (a): air enclosed in a cylinder by a smoothly moving piston of mass 10 kg.

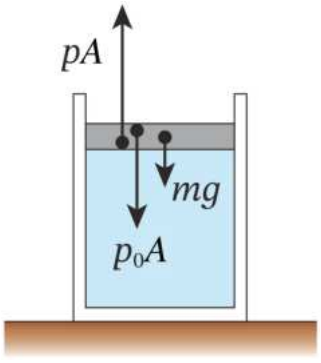

Figure 1-1-5 · Worked Example (a): the balance of forces on the piston — the gas pushes up with pA, while atmospheric p_0A and the weight mg push down.

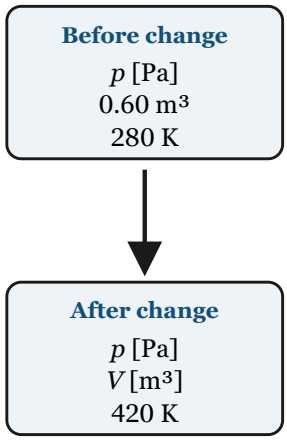

Figure 1-1-6 · Worked Example (c): the gas before and after being heated at constant pressure.

Reasonableness check	(a) The piston weight adds 2.0×10^4 Pa, about one-fifth of an atmosphere, so a small rise above 1.0×10^5 Pa is sensible. (b) The pressure drops sharply while the volume barely changes, so the temperature must fall far below 0°C — a large negative Celsius value fits. (c) Heating from 280 K to 420 K is a factor of 1.5, and 0.60 m^3 × 1.5 = 0.90 m^3.
😳 **Explain it yourself**	In one sentence, say why the mass of the piston raises the gas pressure above atmospheric pressure, and why the temperature must be in kelvin before you substitute it.

✍️ **Now you try:** An ideal gas at 27°C, 1.0×10^5 Pa, and 2.0 L is heated at constant pressure to 87°C. Find its new volume.

🔆 Try

1. Give each answer to 2 significant figures. Air is enclosed in a cylindrical container by a smoothly moving piston with a cross-sectional area of 4.9×10^{-2} m^2 and a mass of 50 kg. Determine the pressure of the air inside the container in Pa. Take the atmospheric pressure as 1.0×10^5 Pa and the acceleration due to gravity as 9.8 m/s^2.
2. An ideal gas at 27°C, 1.5×10^5 Pa, and 0.60 m^3 is changed to 127°C and 0.20 m^3. Determine its pressure in Pa.
3. A gas with a pressure of 2.0×10^5 Pa and a volume of 0.44 m^3 is compressed to 2.2×10^5 Pa while keeping the temperature constant. Determine the volume in m^3.
4. A gas with a volume of 1.0 m^3 and a temperature of 300 K is heated to 360 K while keeping the pressure constant. Determine the volume in m^3.

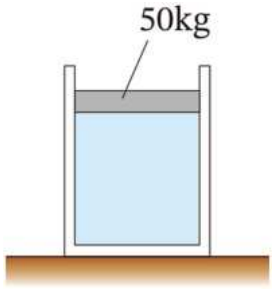

Figure 1-1-7 · Try 1: air enclosed in a cylinder by a smoothly moving piston of cross-sectional area 4.9×10^{-2} m^2 and mass 50 kg.

🌐 In a new context

A weather balloon leaves the ground carrying a fixed mass of helium. As it rises, the outside air pressure falls and the air grows colder, yet the same law pV/T = constant governs the gas the whole way up. Because pressure drops much faster than temperature, the balloon swells until, near the top of its climb, it can reach many times its launch volume — which is why it is filled only loosely at the surface.

➤ **Your turn:** A balloon holds 5.0 m^3 of gas at 1.0×10^5 Pa and 300 K near the ground. High up, the pressure is 2.5×10^4 Pa and the temperature is 250 K. Using pV/T = constant, estimate the new volume.

Figure 1-1-8 · A weather balloon shown at launch, small and slack, and again high in the darkening upper atmosphere where the fall in outside pressure lets its trapped gas expand into a large taut sphere, while the hanging instrument package stays the same size.

🧪 Nature of Science

From the original — bubbles rising from the deep sea and Boyle's law. Bubbles exhaled by a diver grow larger as they rise toward the surface. As the water becomes shallower, the surrounding water pressure decreases, so the gas volume increases in inverse proportion to the pressure. Even the invisible properties of a gas can be described by a physical law.

Boyle-Charles's law is stated for an *ideal* gas — one whose molecular size and intermolecular forces are ignored. This idealization is a scientific model: it is not perfectly true of any real gas, yet it predicts the behavior of ordinary gases closely enough to be trusted, and it fails in a known direction (at very high pressure or very low temperature). Good models are judged by how well and how honestly they predict, not by being exactly true.

Figure 1-1-9 · Bubbles exhaled by a diver grow larger as they rise toward the surface, because the surrounding water pressure falls with decreasing depth.

Your turn: Suggest one everyday situation where a real gas would *not* follow the ideal-gas law well, and say which assumption breaks down.

Lesson question, answered — For a fixed mass of an ideal gas, pressure, volume, and absolute temperature are tied together by pV/T = constant: raising the pressure at fixed temperature shrinks the volume, and raising the absolute temperature at fixed pressure increases it. Pressure itself is force per unit area, $p = F/A$, so a piston's weight adds mg/A to the atmospheric pressure it encloses.

The scene we opened with.

📋 Exercise

1. Give each answer to 2 significant figures. Air is enclosed in a cylindrical container by a smoothly moving piston with a cross-sectional area of 4.9×10^{-3} m^2 and a mass of 20 kg. Determine the pressure of the air inside the container in Pa. Take the atmospheric pressure as 1.0×10^{5} Pa and the acceleration due to gravity as 9.8 m/s^2.
2. Air is enclosed in a cylindrical container by a smoothly moving piston with a cross-sectional area of 1.0 cm^2. When this piston is pushed with a force of 40 N, determine the pressure of the air inside the container in Pa. Take the atmospheric pressure as 1.0×10^{5} Pa.
3. An ideal gas at 27°C, 2.0×10^{5} Pa, and 3.0×10^{-2} m^3 is changed to 87°C and 1.0×10^{5} Pa. Determine its volume in m^3.
4. An ideal gas at 77°C, 3.0×10^{5} Pa, and 500 mL is changed to 47°C and 2.0 L. Determine its pressure in Pa.
5. A gas with a pressure of 1.0×10^{5} Pa and a volume of 0.80 m^3 has its volume changed to 0.40 m^3 while keeping the temperature constant. Determine the pressure in Pa.
6. A gas with a pressure of 2.0×10^{5} Pa and a volume of 3.0×10^{-2} m^3 has its pressure changed to 1.0×10^{5} Pa while keeping the temperature constant. Determine the volume in m^3.
7. A gas with a volume of 1.1×10^{-2} m^3 and a temperature of 330 K is cooled to 300 K while keeping the pressure constant. Determine the volume in m^3.
8. A gas with a volume of 3.0×10^{-2} m^3 and a temperature of 300 K is cooled to 200 K while keeping the pressure constant. Determine the volume in m^3.

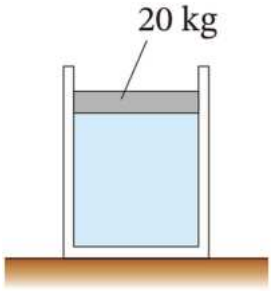

Figure 1-1-10 · Exercise 1: the same cylinder with a piston of cross-sectional area 4.9×10^{-3} m^2 and mass 20 kg.

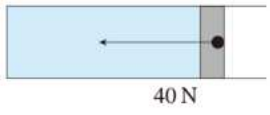

Figure 1-1-11 · Exercise 2: a piston of cross-sectional area 1.0 cm^2 pushed horizontally with a force of 40 N, so gravity does not enter the balance.

1-2 Equation of State for an Ideal Gas

❓ **Lesson question:** What relationship holds among the pressure, volume, temperature, and amount of substance (number of moles) of a gas?

You will learn to

1. State that a collection of 6.02×10^{23} particles is one mole (mol), and use the Avogadro constant to relate moles to the number of molecules.
2. Apply the equation of state for an ideal gas, $pV = nRT$, with the gas constant $R \approx 8.31$ J/(mol·K).
3. Substitute pressure in Pa, volume in m^3, and absolute temperature in K to find any one unknown among p, V, T, and n.

🔍 The phenomenon — predict before you continue

A single helium cylinder can blow up a whole party's worth of balloons before it runs dry. Knowing only the tank's volume, its pressure, and the temperature, you can work out the amount of gas sealed inside — and so roughly how many balloons are left in it — because the pressure, volume, temperature, and amount of substance (in moles) of any gas are locked together by one relation. It makes no difference that the gas is helium: a mole of *any* gas behaves the same way.

★ **Key idea**

Amount of substance n — the gas counted in moles (mol), not in grams. How is n locked to p, V, and T?

Figure 1-2-1 · One helium cylinder fills many party balloons: its pressure, volume, and temperature fix the amount of gas (in moles) it holds.

Predict

(a) To fill twice the volume at the same pressure and temperature, must there be more, less, or the same amount of substance (moles) of gas?

(b) In a rigid container at fixed temperature, if the pressure reads higher, is there more, less, or the same amount of substance (moles) of gas inside?

🔍 Explore — Warming sealed air at fixed volume

What you'll do: heat a fixed sample of air trapped in a rigid flask and track how its pressure and temperature rise together, then test whether pV/T stays fixed.

Time: 15 min

You will need:

- a rigid sealed flask of known volume V [L]
- a pressure gauge
- a thermometer
- a water bath

rigid flask

pressure gauge

thermometer

water bath

Steps

1. Record the flask's fixed inside volume V in liters (L) once, since it stays the same for every run.
2. At room temperature read the pressure p from the gauge and the temperature; if the gauge reads gauge pressure, add the atmospheric pressure (about 100 kPa) to get the absolute p. Then work out the absolute temperature T [K] = T [°C] + 273.
3. Stand the flask in the water bath, warm it gently, and record a new p and T pair at each step while the volume stays fixed.
4. For each run work out pV/T and enter it in the last column.

Run	Temperature T (K)	Pressure p (kPa)	pV/T (J/K, with V in L)
1			
2			
3			

Analyze what you collected:

(a) As you warm the gas at fixed volume, how do the pressure p and the absolute temperature T change together? Look down the last column — what do you notice about pV/T across the runs?

(b) The quantity pV/T equals nR, and R is the same for every gas. What does a steady pV/T tell you about the amount of substance n sealed inside? Divide your column value by R = 8.3 J/(mol·K) to predict how many moles the flask holds.

✎ Write your explanation in two lines before you read the next page.

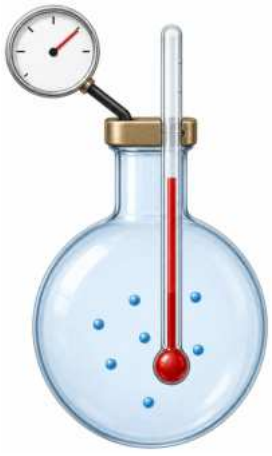
Figure 1-2-2 · A rigid sealed flask of fixed volume fitted with a pressure gauge and a thermometer.

★ Working note

Keep the volume truly fixed by using a rigid flask, let the gauge and the thermometer settle at each temperature before you read them, and convert every temperature to kelvin with T [K] = T [°C] + 273. With p in kPa and V in L the product pV is already in joules, because 1 kPa × 1 L = 1 J.

From observation to model

The mole and the Avogadro constant

Molecules are far too many and far too small to count one at a time, so we count them in packages instead. The mole is that package, and it is sized deliberately: a sample you can weigh on a bench balance holds a convenient number of moles. The Avogadro constant N_A is the exchange rate between the two worlds — multiply an amount in moles by N_A to get the actual number of molecules, divide to come back. This is why gas laws are written in moles: it is the only way to put a countable number into an equation.

The standard state and the equation of state

Start from your own last column. The pressure and the temperature both climbed as you warmed the flask, yet pV/T held one value across every run — which is what the previous lesson found for a fixed mass of gas. But that value belongs to your flask, not to gases in general: seal in twice as much gas at the same pressure and temperature and it fills twice the volume, so pV/T doubles with it. The constant is therefore proportional to the amount of substance,

$$\frac{pV}{T} = nR$$

with R the same for every sample; and multiplying through by T puts the whole relation on one line:

$$pV = nRT$$

Only the size of R is still open, and a single measurement closes it, since $R = pV/(nT)$ needs one sample whose four readings are all known. That is what a reference state is for — a volume means nothing until the pressure and the temperature beside it are agreed on — and in the standard state all four are: one mole, 273 K, one atmosphere, and the 22.4 L that mole is measured to fill. The striking part is that the same 22.4 L comes back whichever gas is weighed out, because the ideal model has thrown away the size of the molecules and the forces between them and left nothing but how many there are. So the division returns one and the same 8.31 J/(mol·K) for helium as for nitrogen, and that is what makes R a constant of nature rather than a property of any one gas.

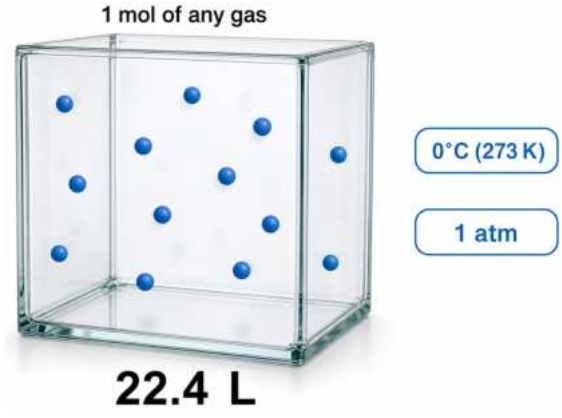

Figure 1-2-3 · One mole of any ideal gas fills 22.4 L in the standard state, 0°C and 1 atm.

★ **Key fact**

How big is a mole? If you could count a million molecules every second, counting one mole would still take about twenty billion years — longer than the universe has existed.

★ **Term**

Amount of substance — a count, not a mass. Two grams of hydrogen and two grams of oxygen weigh the same but hold very different numbers of molecules, which is why the gas laws are written in moles rather than in grams.

★ **Note**

More than one reference condition is in use — chemistry often quotes 25°C instead of 0°C — and each gives a different molar volume. Check which one a question intends before you reach for 22.4 L.

Point

- A collection of 6.02×10^{23} particles is expressed as 1 **mole** (symbol **mol**), and the quantity expressed using the unit mol is called the **amount of substance**. The amount 6.02×10^{23}/mol is called the **Avogadro constant**.
- The state of a gas at a temperature of **0°C (273 K)** and a pressure of **1 atm** is called the **standard state**. In the standard state, regardless of the type of gas, the volume of a gas per 1 mol is **22.4** L.
- For an ideal gas with amount of substance n [mol], pressure p [Pa], volume V [m^3], and temperature T [K], **$pV = nRT$** holds. This is called the **equation of state for an ideal gas**. This R is called the gas constant, and regardless of the type of gas, it is approximately 8.31 J/(mol·K).

Equation box

Equation of state	$pV = nRT$ (p in Pa, V in m^3, T in K, n in mol)

Gas constant	$R \approx 8.31$ J/(mol·K), the same for any gas
Standard molar volume	1 mol of an ideal gas occupies 22.4 L at 0°C (273 K) and 1 atm
Temperature	T [K] = T [°C] + 273.

🛠 Worked Example

Situation	Determine the amount of substance in moles of an ideal gas with a pressure of 1.0×10^{5} Pa, a temperature of 27°C, and a volume of 2.0×10^{-3} m^3, to 2 significant figures. Take the gas constant as 8.3 J/(mol·K).
Given and required	$p = 1.0\times10^{5}$ Pa, T [°C] = 27°C so $T = 27 + 273 = 300$ K, $V = 2.0\times10^{-3}$ m^3, $R = 8.3$ J/(mol·K); find the amount of substance n [mol].
Representation	The gas is a fixed sample described by a single state (p, V, T). See Figure 1-2-2 for the flask, gauge, and thermometer that fix these three readings.
Strategy	Steps to apply the ideal gas equation: check the given pressure, volume, absolute temperature, and number of moles of the gas, and substitute them into the equation of state $pV = nRT$ to determine the unknown quantity.
Annotated solution	Let the required amount of substance be n [mol]. From the equation of state for an ideal gas $pV = nRT$: $(1.0\times10^{5})\times(2.0\times10^{-3}) = n\times8.3\times(27+273)$ $200 = n\times8.3\times300 = n\times2490$ $n = 200 / 2490 =$ **8.0×10^{-2} mol**.
Reasonableness check	At 0°C and 1 atm one mole fills 22.4 L. Here the volume is only 2.0 L (2.0×10^{-3} m^3) and the pressure is close to 1 atm, so well under one mole is expected — and 8.0×10^{-2} mol is a small fraction of a mole, which fits.
🧐 Explain it yourself	In one sentence, say why the Celsius temperature must be converted to kelvin before it is substituted into $pV = nRT$.

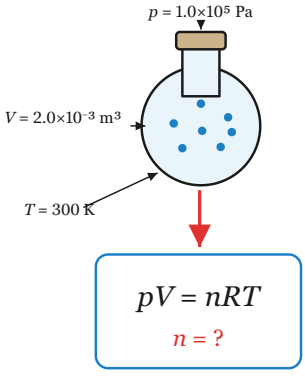

Figure 1-2-4 · The known p, V, and T are substituted into pV = nRT to find n.

✍ **Now you try:** Determine the amount of substance in moles of an ideal gas with a pressure of 1.0×10^{5} Pa, a temperature of 27°C, and a volume of 4.0×10^{-3} m^3, to 2 significant figures. Take $R = 8.3$ J/(mol·K).

🔆 Try

1. Determine the volume in m^3 of an ideal gas with a pressure of 1.2×10^{5} Pa, a temperature of 373 K, and an amount of substance of 2.0 mol, to 2 significant figures. Take the gas constant as 8.3 J/(mol·K).

2. Determine the amount of substance in moles of an ideal gas with a pressure of 1.7×10^{5} Pa, a temperature of 27°C, and a volume of 6.0×10^{-3} m^3, to 2 significant figures. Take the gas constant as 8.3 J/(mol·K).

🌐 In a new context

A rigid steel scuba tank holds a fixed sample of air. Because its volume is set by the metal wall, the equation of state $pV = nRT$ lets a diver read the tank's pressure gauge and, knowing the tank volume and the temperature, work out how many moles of air remain — and therefore how many breaths are left. As the tank empties, n falls, so at fixed V and T the pressure p drops in step with the amount of gas still inside.

➤ **Your turn:** A rigid 1.2×10^{-2} m^3 tank holds gas at 8.31×10^{6} Pa and 300 K. Using $pV = nRT$ with R = 8.3 J/(mol·K), estimate the amount of substance inside.

Figure 1-2-5 · A rigid steel scuba cylinder holds a fixed volume of densely packed air molecules at high pressure, while the diver's exhaled bubbles grow larger as they rise toward the surface and the surrounding pressure falls.

Figure 1-2-6 · About eleven large plastic bottles hold the same 22.4 L as one mole of gas, some 6.02×10^{23} molecules.

🧪 Nature of Science

From the original — the tremendously large number "Avogadro constant." The amount of substance (moles) that appears in the ideal gas equation is a unit for counting the number of gas molecules. The number of molecules contained in one mole is called the Avogadro constant, about 6.02×10^{23} — a tremendously large number. An ideal gas in the standard state (0°C, 1 atm) occupies 22.4 L, about eleven large plastic bottles, regardless of its type, and inside it such an astronomical number of molecules are flying around.

The gas constant R is one number that works for every gas. That a single measured constant captures the behavior of oxygen, helium, and air alike is a hallmark of a good physical model: it compresses many separate observations into one relation, $pV = nRT$, that can then be tested against fresh measurements.

Your turn: Suggest one way you could measure the Avogadro constant, or the gas constant R, from readings you could take in a school laboratory.

Lesson question, answered — For an ideal gas the pressure, volume, temperature, and amount of substance are tied together by one equation of state, $pV = nRT$, where $R \approx 8.31$ J/(mol·K) is the same for every gas. The amount of substance n counts molecules in moles, with 6.02×10^{23} molecules per mole, and any one of the four quantities is fixed once the other three are known.

The scene we opened with.

📋 Exercise

1. Determine the volume in m^3 of an ideal gas with a pressure of 1.2×10^{5} Pa, a temperature of 27°C, and an amount of substance of 2.0 mol, to 2 significant figures. Take the gas constant as 8.3 J/(mol·K).
2. Determine the temperature in °C of an ideal gas with a pressure of 4.0×10^{4} Pa, an amount of substance of 0.50 mol, and a volume of 3.32×10^{-2} m^3, to 2 significant figures. Take the gas constant as 8.3 J/(mol·K).
3. Determine the amount of substance in moles of an ideal gas with a pressure of 3.0×10^{5} Pa, a temperature of 127°C, and a volume of 1.66×10^{-2} m^3, to 2 significant figures. Take the gas constant as 8.3 J/(mol·K).

1-3 Motion of Gas Molecules

❓ **Lesson question:** From a microscopic view, what motion of gas molecules produces the pressure of a gas?

You will learn to

1. Find the impulse one molecule delivers to a wall in a single elastic collision, $2mv_x$, and count how often it strikes the same wall.
2. Build up from one molecule to N molecules to obtain the average force F and the pressure p on a wall.
3. Derive the kinetic relation $p = Nm\overline{v^2}/3V$ and read it as a microscopic origin of pressure.

🔍 The phenomenon — predict before you continue

Let a wisp of smoke drift into a small glass cell and watch it under a microscope. The bright specks never settle: each one jitters and darts along a broken, random path, as though it were being shoved from every side at once. Nothing visible touches them — they are being struck, unevenly and ceaselessly, by the air molecules around them. Those same countless impacts, delivered to a wall instead of to a speck, add up to the steady push we call pressure.

★ **Key idea**

Impulse — the change of momentum a body receives, in newton-seconds [N·s]. A gas holds N molecules of mass m at speed v in a volume V. What must its walls feel?

Figure 1-3-1 · Smoke specks under a microscope never rest: air molecules strike them unevenly from every side, and those same countless impacts, spread over a wall, make up pressure.

Predict

(a) Does one molecule push on the wall all the time, or only at the instant it hits?

(b) If you doubled the number of molecules while keeping their speed the same, would the average force on the wall roughly double, halve, or stay the same?

🔍 Explore — Beads in a shaken box

What you'll do: Reason through a mechanical model of a gas — many beads rattling in a shaken box against one light, freely sliding wall — to see how countless separate impacts add up to the one steady push we call pressure.

Time: 8 min

You will need:

- A small clear rigid box
- A few dozen identical beads
- A light card that slides freely as one wall
- A ruler

Clear box

Beads

Sliding card

Ruler

Steps

1. Picture N identical beads of mass m rattling in the box and striking the sliding wall with speed v; each elastic hit reverses the bead, so it hands the wall a momentum of about $2mv$.
2. Reason about how often one bead comes back to the *same* wall: a faster bead makes its round trip sooner, so its hits per second grow with v.
3. For each model run, multiply momentum-per-hit by hits-per-second to get the average force on the wall, and enter it in the last column.
4. Read down that last column and compare the runs to see how the force answers to *more beads* and to *faster beads*.

> ★ **Working note**
> Each bead touches the wall only for an instant, yet the hits arrive so often that the wall feels a smooth, constant push. Track two things for each wall — how *hard* one hit is (the momentum it hands over) and how *often* the hits come (hits per second) — and their product is the average force.

Model run	Momentum per hit ($\propto mv$)	Hits per second ($\propto v$)	Average force on wall ($\propto Nmv^2$)
N beads, speed v (baseline)			
$2N$ beads, speed v			
N beads, speed $2v$			

Analyze what you reasoned out:

(a) Reading down the force column: how does doubling the number of beads change the push, and how does doubling their speed change it? Which change is steeper, and why do both the harder hit *and* the more frequent hit grow with speed?

(b) A real gas wall behaves the same way. Predict how the gas pressure should depend on the number of molecules N and on their speed — and check your prediction against $p = Nm\overline{v^2}/3V$.

✎ Write your explanation in two lines before you read the next page.

From observation to model

One collision gives an impulse of $2mv_x$

Your bead runs have already fixed the shape of the answer: twice as many beads doubled the push, while twice the speed quadrupled it, because a faster bead both hits harder and comes back sooner. What is missing is a number in place of a

proportionality, and that means following one bead through one hit. Consider an ideal gas of N molecules, each of mass m [kg], in a cubic container of side L [m] and volume $V = L^3$ [m³]. Look at the surface S perpendicular to the x-axis. Because the collision with the wall is elastic, only the velocity component perpendicular to the wall reverses: the x-component changes from v_x to $-v_x$. The change in the molecule's momentum along x is $(-mv_x) - mv_x = -2mv_x$, so the wall pushes the molecule with an impulse of magnitude $2mv_x$. By the law of action and reaction, the molecule delivers an impulse of $2mv_x$ [N·s] to the wall.

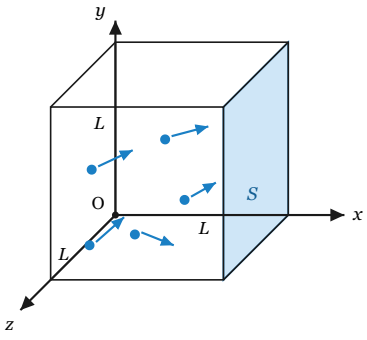

Figure 1-3-2 · Cubic container of side L with axes x, y, z and the shaded face $S \perp x$; gas molecules move along the velocity arrows.

Many impacts average into a steady force and a pressure

Along the x-axis the molecule moves back and forth at constant speed v_x, covering a round-trip distance $2L$ between successive hits on S, so it strikes S once every $2L/v_x$ [s]. In a time t it therefore strikes $v_x t/2L$ times, delivering a total impulse $mv_x^2 t/L$; dividing by t gives the average force from one molecule,

$$f = \frac{mv_x^2}{L}$$

Summing over all N of them replaces the single v_x^2 by its average, so the force on S is $F = Nm\overline{v_x^2}/L$; the face has area L^2, so $p = F/L^2 = Nm\overline{v_x^2}/L^3 = Nm\overline{v_x^2}/V$. One step is left, and it is the step that removes the axis we happened to choose. The motion is random with no preferred direction, so the three components carry equal shares, $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$; and since $v^2 = v_x^2 + v_y^2 + v_z^2$, each share is a third of the whole, $\overline{v_x^2} = \overline{v^2}/3$. Substituting it gives

$$p = \frac{Nm\overline{v^2}}{3V}$$

— a pressure fixed by how many molecules there are, how heavy each is, and how fast they move, and by nothing else. Nothing in it remembers which face we watched, and nothing in it mentions the shape of the box.

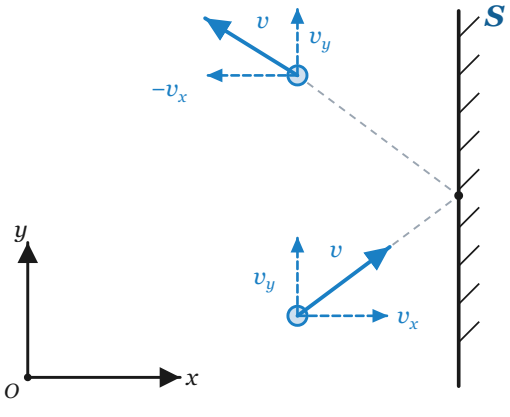

Figure 1-3-3 · Elastic collision at S: the perpendicular component reverses ($v_x \to -v_x$) while the parallel component v_y is unchanged, so the speed v is preserved.

★ **Key fact**
Impulse = change in momentum. One elastic hit reverses v_x, giving the wall an impulse of $2mv_x$ [N·s].

★ **Term**
Mean square speed $\overline{v^2}$ — the average of v^2 over all molecules. Its square root $\sqrt{\overline{v^2}}$ is the root-mean-square speed.

★ **Note**
Random motion has no preferred direction, so $\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$ and $\overline{v^2} = 3\overline{v_x^2}$.

Point

Steps to determine the pressure from the motion of gas molecules

1. Determine the impulse exerted on the wall by one molecule in a single collision.
2. Determine the total impulse exerted on the wall by one molecule during time t.
3. Determine the average force exerted on the wall by one molecule.
4. Determine the average force exerted on the wall by N molecules.
5. Determine the pressure of the gas.

Equation box

One collision	impulse on wall = $2mv_x$ (unit: N·s)
One molecule	hits per time $t = \dfrac{v_x t}{2L} \Rightarrow f = \dfrac{mv_x^2}{L}$
N molecules	$F = \dfrac{Nm\overline{v_x^2}}{L}$ and $p = \dfrac{F}{L^2} = \dfrac{Nm\overline{v_x^2}}{V}$
Kinetic pressure	$\overline{v^2} = 3\overline{v_x^2} \Rightarrow p = \dfrac{Nm\overline{v^2}}{3V}$ (unit: Pa)
Density form	$p = \dfrac{Nm\bar{v}^2}{3V} = \frac{1}{3}\rho\bar{v}^2$

Worked Example

Situation

An ideal gas of N molecules of mass m [kg] fills a cubic container of side L [m] and volume $V = L^3$ [m^3]. With x, y, z axes set up, consider the pressure on the surface S perpendicular to the x-axis. The molecules do not collide with each other and move in straight lines at constant speed until they strike the walls; collisions with the walls are elastic, so the speeds are unchanged. Let one molecule have speed v [m/s] with components v_x, v_y, v_z.

(a) Find the magnitude of the impulse given to S by a single collision of one molecule.

(b) Find the magnitude of the impulse given to S by one molecule during time t [s].

(c) Find the average force f [N] exerted on S by one molecule.

(d) Find the average force F [N] exerted on S by N molecules, letting the average of v_x^2 be $\overline{v_x^2}$.

(e) Express the pressure p [Pa] in terms of V [m^3] and the average of the squared speeds $\overline{v^2}$.

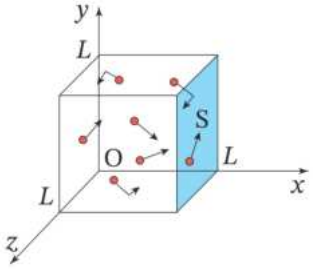

Figure 1-3-4 · Worked Example: molecules move in a cube of side L; each collision with the shaded wall S reverses the x-component of a molecule's momentum.

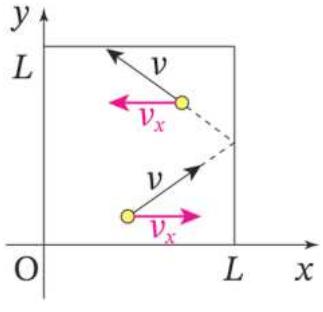

Figure 1-3-5 · Worked Example: a molecule crosses the box of side L and rebounds from the face at $x = L$, so v_x reverses while the speed v is unchanged.

Given and required	Given: N, m, cubic side L, $V = L^3$, elastic wall collisions, velocity components v_x, v_y, v_z. Required: (a) impulse per collision on S; (b) impulse on S in time t; (c) f; (d) F using $\overline{v_x^2}$; (e) p in terms of V and $\overline{v^2}$.
Representation	See Figure 1-3-2 for the cube and surface S, Figure 1-3-3 for the reversal of v_x at the wall, and Figure 1-3-4 for the round trip of length $2L$ along the x-axis.
Strategy	Work through the five steps of the Point in order, and note what each one needs: the impulse comes from the change in momentum, the collision rate from the round-trip distance $2L$, and the last step from isotropy — no direction is special, so $\overline{v^2} = 3\overline{v_x^2}$.
Annotated solution	**(a)** Elastic collision reverses only the x-component: $v_x \to -v_x$. Momentum change along x is $(-mv_x) - mv_x = -2mv_x$; the wall's impulse on the molecule is $-2mv_x$, so by action–reaction the impulse on S is $\boldsymbol{2mv_x}$ **[N·s]**. **(b)** Along x the motion is uniform at speed v_x; the round trip to S is $2L$, so the time between hits is $2L/v_x$ and the number of hits in t is $t \div (2L/v_x) = v_x t/2L$. With $2mv_x$ per hit, the total impulse on S is $2mv_x \times v_x t/2L =$ $\boldsymbol{mv_x^2 t/L}$ **[N·s]**. **(c)** This impulse equals ft: $ft = mv_x^2 t/L$. Dividing by t, $\boldsymbol{f = mv_x^2/L}$ **[N]**. **(d)** With the average $\overline{v_x^2}$ over all molecules, from (c) the force from N molecules is $F = N \times m\overline{v_x^2}/L =$ $\boldsymbol{Nm\overline{v_x^2}/L}$ **[N]**. **(e)** Pressure = $F \div$ (area of $S = L^2$): $p = F/L^2 = Nm\overline{v_x^2}/L^3$. Since $L^3 = V$, $p = Nm\overline{v_x^2}/V$ … (i). Because $\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}$ and the motion is unbiased ($\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}$), $\overline{v^2} = 3\overline{v_x^2}$, i.e. $\overline{v_x^2} = (1/3)\overline{v^2}$. Substituting into (i): $\boldsymbol{p = Nm\overline{v^2}/3V}$ **[Pa]**.
Reasonableness check	The units check: $(2mv_x)$ has kg·m/s = N·s; $Nm\overline{v^2}/V$ has kg·(m/s)²/m³ = (kg·m/s²)/m² = N/m² = Pa. Pressure rises with more molecules N, with faster molecules $\overline{v^2}$, and falls as the volume V grows — all as expected for a gas.
Explain it yourself	In one sentence, say why the factor 1/3 appears in $p = Nm\overline{v^2}/3V$, referring to the three equal direction averages.

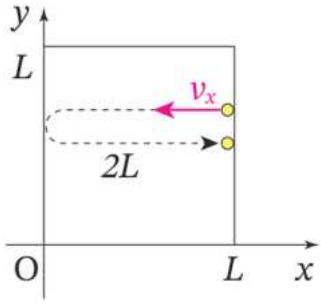

Figure 1-3-6 · Worked Example: between two hits on S the molecule covers a round trip of $2L$, so it strikes S once every $2L/v_x$.

✍ **Now you try:** A molecule of mass $m = 5.0\times10^{-26}$ kg has an x-velocity component $v_x = 400$ m/s inside a cube of side $L = 0.10$ m. Find (a) the impulse it

gives S per collision, (b) how many times per second it strikes S, and (c) the average force it exerts on S.

Try

1. An ideal gas of N molecules of mass m [kg] fills a rectangular box with side lengths L_x [m], L_y [m], L_z [m] and volume $V = L_x \times L_y \times L_z$ [m^3]. With x, y, z axes set up, consider the pressure on the surface S perpendicular to the x-axis. The molecules do not collide with each other, move in straight lines at constant speed, and collide elastically with the walls so their speeds are unchanged. Let one molecule have speed v [m/s] with components v_x, v_y, v_z.
   (a) Find the magnitude of the impulse given to S by a single collision of one molecule.
   (b) Find the magnitude of the impulse given to S by one molecule during time t [s].
   (c) Find the average force f [N] exerted on S by one molecule.
   (d) Find the average force F [N] exerted on S by N molecules, letting the average of v_x^2 be $\overline{v_x^2}$.
   (e) Express the pressure p [Pa] in terms of V [m^3] and the average of the squared speeds $\overline{v^2}$.

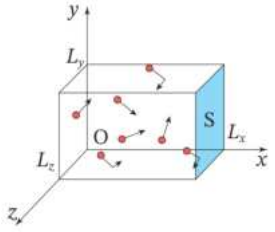

Figure 1-3-7 · Try 1: the same argument in a rectangular box of sides L_x, L_y, L_z, with the face S perpendicular to the x-axis.

In a new context

The air standing still in your room is not still at all: its molecules are hurtling past one another at roughly the speed of a pistol bullet — and two numbers anyone can measure are enough to prove it. Weigh a known volume of air to get its density, about $\rho \approx 1.3$ kg/m^3 at room temperature, and read the pressure from a barometer, about $p \approx 1.0\times10^5$ Pa. Writing the kinetic result $p = Nm\overline{v^2}/3V$ in terms of that density gives $p = \tfrac{1}{3}\rho\overline{v^2}$, so those two everyday, large-scale readings alone fix the speed of particles far too small to see.

➤ **Your turn:** Using $p = \tfrac{1}{3}\rho\overline{v^2}$ with $\rho = 1.3$ kg/m^3 and $p = 1.0\times10^5$ Pa, estimate the root-mean-square speed $\sqrt{\overline{v^2}}$ of air molecules.

Nature of Science

From the original — gas molecules flying around like bullets. The air pressure we feel is caused by countless molecules constantly colliding with walls and our skin. At room temperature the average speed of air molecules such as nitrogen and oxygen reaches about 500 meters per second — faster than the speed of sound and comparable to a pistol bullet. The estimate above, $\approx 4.8\times10^2$ m/s, is the root-mean-square speed and is of the same order as this average, reproducing it from the kinetic model, a striking match between a microscopic picture and a macroscopic measurement.

The derivation is a chain of idealizations: point-like molecules, no collisions between molecules, perfectly elastic walls, and a smooth average replacing many separate impacts. None is exactly true, yet together they turn an impossible bookkeeping of trillions of impacts into one clean equation that predicts real pressures well. This is how kinetic theory works — it links the invisible motion of molecules to a quantity we can measure with a gauge.

Daniel Bernoulli set out this very argument in 1738 — pressure as the drumming of countless particles — and it was left almost untouched for more than a century, until Clausius in 1857 and Maxwell in 1860 rebuilt it and made it

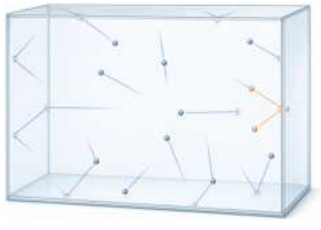

Figure 1-3-8 · Air molecules fly like tiny bullets at about 500 m/s and rebound elastically off the wall; each impact is one of the countless collisions that make up the pressure.

calculate. A model can be right and still wait a hundred years for the evidence that makes it worth believing.

Your turn: Suggest one measurement you could make on a real gas that would test whether the model's prediction $p = Nm\overline{v^2}/3V$ holds, and name the assumption most likely to break down.

Lesson question, answered — Gas pressure comes from the ceaseless, straight-line motion of molecules that strike the container walls and rebound elastically. Each hit delivers a tiny impulse $2mv_x$, and the impacts are so frequent that, averaged over N molecules and all directions, they act as a steady force per unit area: $p = Nm\overline{v^2}/3V$. Pressure is the large-scale signature of molecular speed.

The scene we opened with.

📋 Exercise

1. One mole of gas molecules of mass m [kg] is placed in a cubic container of side L [m] and volume $V = L^3$ [m^3]. The molecules do not collide with each other and collide elastically with the surfaces. With x, y, z axes set up, consider the pressure on the surface S perpendicular to the x-axis. Let the velocity components of a molecule be v_x, v_y, v_z. Fill in each blank with the appropriate expression.

   In one collision with S, the impulse this molecule gives S is (A), and the number of times it strikes S per second is (B). So the total impulse it gives S per second is (C). Let the Avogadro constant be N_A and the average of v_x^2 be $\overline{v_x^2}$. Then the total impulse given to S by all molecules per second is (D). If F is the average force on S from all molecules, then F = (E), and using $\overline{v_x^2} = (1/3)\overline{v^2}$, the pressure of the gas is p = (F).

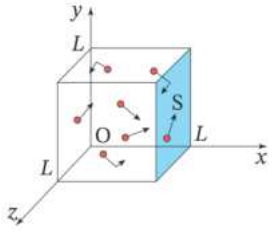

Figure 1-3-9 · Exercise: one mole of gas in a cubic container of side L, with the surface S perpendicular to the x-axis.

1-4 Internal Energy of a Gas

❓ **Lesson question:** What determines the "internal energy" stored in a gas?

You will learn to

1. Work out how much energy a single-atom gas holds, in joules (J), from nothing but its amount and its absolute temperature.
2. Find how that store shifts when the same sample is warmed or cooled through a stated number of kelvin.
3. Predict the settled temperature and pressure when two insulated vessels are opened to each other and their gases mix.

🔍 The phenomenon — predict before you continue

Two rigid, well-insulated flasks hold the same monatomic gas, helium. One flask is warm and one is cool, and they are joined by a closed valve. Nothing can enter or leave, and no heat escapes through the insulation. When the valve is opened, the two bodies of gas mix and settle to a single steady temperature. No stove and no ice were used — only the gas the flasks already held.

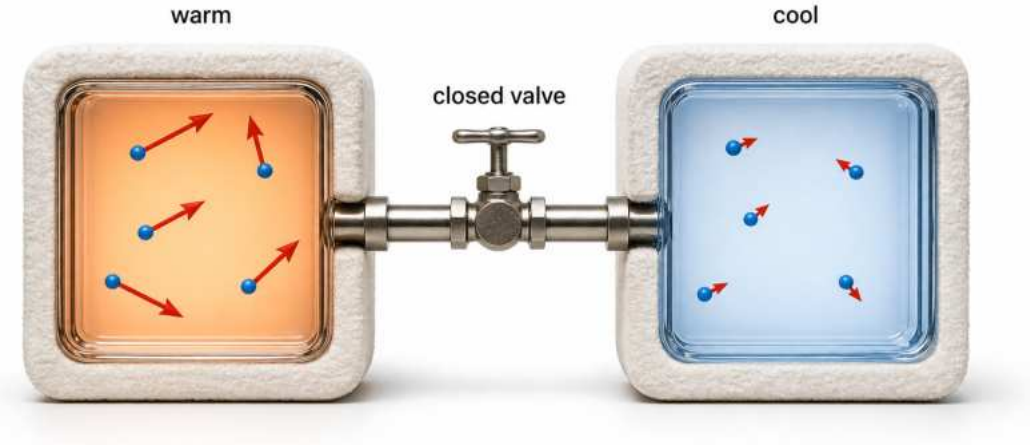

Figure 1-4-1 · Two insulated flasks of helium, one warm and one cool, joined by a closed valve; the warmer atoms move faster.

Predict

(a) After the valve is opened, is the final temperature above the warmer gas, below the cooler gas, or somewhere between the two?

(b) Does the total internal energy of the gas in the two flasks rise, fall, or stay the same?

★ **Key idea**

Internal energy U — the sum of the kinetic energies of all the molecules of a gas and the potential energy of the intermolecular forces, measured in joules (J); in a monatomic ideal gas the forces vanish, so only the kinetic part is left.

🔍 Explore — Mixing warm and cool air

What you'll do: Let two insulated bottles of air at different temperatures share their gas, and test whether the settled temperature is the average of the two starts — the sign that internal energy is conserved.

Time: 15 min

You will need:

- Two rigid insulated bottles (A and B), same size
- Two thermometers
- A warm water bath
- A clamped connecting tube and insulating wrap

Insulated bottle

Thermometer

Water bath

Steps

1. Trap equal amounts of air in the two rigid insulated bottles, fit a thermometer through each stopper, and note the room-temperature reading of each.
2. Stand bottle A in the warm water bath until its reading is well above room temperature; leave bottle B cool. Record the starting temperatures T_A and T_B.
3. Wrap both bottles and the tube, open the clamp, and record the single temperature T_{mix} the pair settles to; then compute the predicted average $\frac{1}{2}(T_A+T_B)$.
4. Repeat with a larger gap between T_A and T_B.

Run	T_A warm /°C	T_B cool /°C	T_{mix} settled /°C	$\frac{1}{2}(T_A+T_B)$ /°C
1				
2				
3				

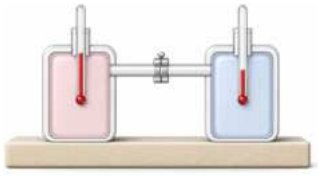

Figure 1-4-2 · Two insulated bottles of trapped air (A warmed, B cool), each read by a thermometer, joined by a clamped tube on a foam stand; opening the clamp lets them mix.

> ★ **Working note**
> Keep each thermometer bulb in the trapped air, not touching the glass wall, and read a temperature only after it stops drifting — the bottles reach a steady value slowly through the insulation.

Analyze what you collected:

(a) Across the runs, how does the settled temperature T_{mix} compare with your predicted average $\frac{1}{2}(T_A+T_B)$?

(b) The bottles are rigid (the gas does no work) and insulated (no heat leaves). What does the match tell you about the total internal energy of the two gases, and where did the warm gas's lost energy go?

✎ Write your explanation in two lines before you read the next page.

From observation to model

Where a gas keeps its energy

A gas can hold energy in only two places. One is motion: every molecule is flying, so it carries kinetic energy, and that share grows as the gas gets hotter. The other is position: molecules pull on one another across the gaps between them, and moving them nearer or farther changes the stored energy of that attraction, just as lifting a magnet away from iron does. Both shares belong to the gas itself and neither can be seen from outside, which is why they are counted together as one bookkeeping quantity. Helium, neon and argon are the simplest case to count, because a lone atom has no bonds inside it to stretch and, in the ideal limit, its neighbors are too far

away for their attraction to register — the second store is empty, and only the flying is left to add up.

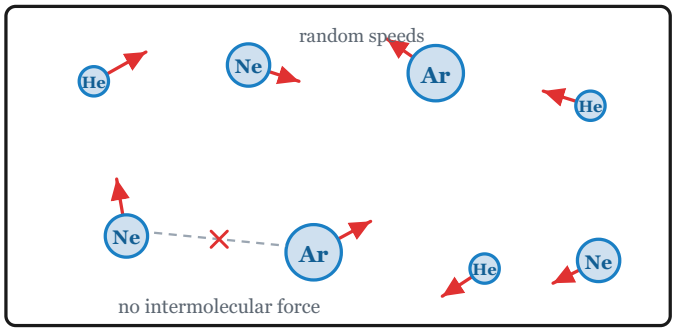

Figure 1-4-3 · Monatomic molecules — single atoms of He, Ne, and Ar — moving in random directions at random speeds inside a box, with no intermolecular force acting between them.

Deriving the (3/2) — and why nothing else survives

Two results already in hand meet here. Lesson 1-3 built the pressure out of molecular impacts, $p = Nm\overline{v^2}/3V$, which rearranges to $pV = \frac{1}{3}Nm\overline{v^2}$; the equation of state describes the same sample from outside as $pV = nRT$. Equating them gives

$$\frac{1}{3}Nm\overline{v^2} = nRT$$

and since the total kinetic energy of the sample is $\frac{1}{2}Nm\overline{v^2}$, i.e. (3/2) times the left-hand side, that total is exactly $(3/2)nRT$. The derivation is worth reading twice for what it loses along the way: the molecular mass m, the speed v, the volume and the chemical identity of the gas all cancel out. Two flasks of different monatomic gases at the same amount and the same temperature therefore hold the same energy, and warming a sample by ΔT costs the same whatever gas it is.

What follows when nothing gets in or out

Close a gas inside walls that cannot move and cannot pass heat — a wall that passes no heat is called **adiabatic**. Nothing can push the boundary, so no work crosses it; nothing can flow through it, so no heat crosses it either. Energy reaches a trapped gas only as heat or as work — all that the first law of thermodynamics, set out in Lesson 1-5, asserts — so its energy account has no entry left to make. Join two such vessels and the same argument covers both at once: what one body of gas gives up the other must take on, so the two energies simply add to the value they had before. Writing that sum with $(3/2)nR$ in front of every term lets the common factor be struck out, and the settled temperature turns out to be the average of the two starting temperatures weighted by how many moles each vessel brought — the larger sample pulls the result toward itself, which is the calculation carried out in the Example below.

Point

- Molecules consisting of a single atom, such as helium (He), neon (Ne), and argon (Ar), are called **monatomic molecules**.
- The sum of the kinetic energies of the gas molecules and the potential energy due to intermolecular forces is called the **internal energy** of the gas. In a monatomic ideal gas, since there are no intermolecular forces, the internal energy of the gas is simply the sum of its kinetic energies.
- The internal energy U[J] of n[mol] of a monatomic ideal gas at temperature T[K] is expressed by the following equation.

★ **Term**

Where the monatomic model stops — a two-atom molecule such as O_2 can also spin end-over-end and vibrate along its bond, and each of those extra ways of moving takes its own share of energy. So a diatomic or polyatomic gas stores more than a single atom does at the same temperature, and the (3/2) factor of this lesson is no longer the right one for it. **Where the ideal limit stops** — real atoms do attract one another, so the position share is never exactly zero. Squeeze a real gas hard or cool it toward liquefaction and the molecules spend their time close together: the stored energy then depends on the volume as well as on the temperature, and the gas is on its way to becoming a liquid.

★ **Key fact**

The scale of it: one mole at room temperature, 300 K, carries $(3/2)\times1\times8.3\times300 \approx 3.7\times10^3$ J — roughly what it takes to lift a 5 kg mass 75 m. Halve the kelvin reading and you halve that store.

$U = (\frac{3}{2})nRT$ (R is the gas constant)

Furthermore, the change in internal energy ΔU[J] when the temperature of a gas changes by ΔT[K] is given by $\Delta U = (\frac{3}{2})nR\Delta T$.

! According to the first law of thermodynamics, for a gas in a container of constant volume, when there is no heat exchange with the surroundings, the internal energy of the gas is conserved.

Equation box

Internal energy	$U = (\frac{3}{2})nRT$ (monatomic ideal gas; R is the gas constant)
Change in internal energy	$\Delta U = (\frac{3}{2})nR\Delta T$
Equation of state	$pV = nRT$
Conservation on mixing (adiabatic)	$(\frac{3}{2})n_A RT_A + (\frac{3}{2})n_B RT_B = (\frac{3}{2})(n_A+n_B)RT$
Gas constant	$R \approx 8.31$ J/(mol·K)

★ Note

Kelvin, always: T in this equation is the absolute temperature, so a Celsius reading must be converted before it is substituted. The one exception is a *difference*: a rise of 10 °C is a rise of 10 K, so ΔT may be taken straight from a Celsius change.

🛠 Worked Example

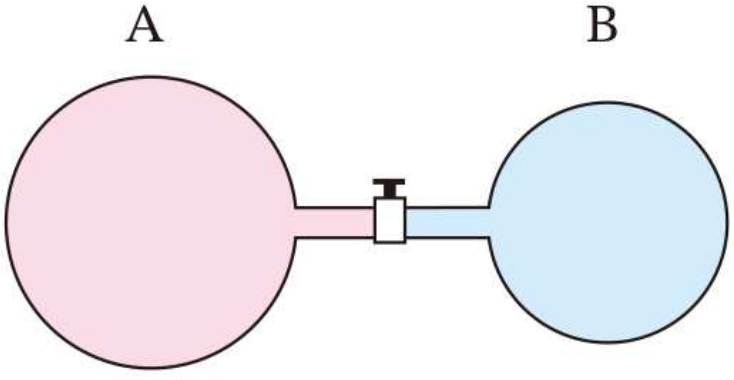

Figure 1-4-4 · Worked Example: the adiabatic containers A and B joined by a thin tube with a stopcock, before any values are entered.

Situation	An adiabatic container A with a volume of 0.62 m^3 and an adiabatic container B with a volume of 0.21 m^3 are connected by a thin tube with a stopcock. With the stopcock closed, 1.5 mol of a monatomic ideal gas at 300 K is enclosed in container A, and 1.0 mol of the gas at 400 K is enclosed in container B. Answer to 2 significant figures for the state after the stopcock is opened and a sufficient amount of time has passed. Take the gas constant as 8.3 J/(mol·K). (a) Find the final temperature of the gas in K. (b) Find the final pressure of the gas in Pa.

Given and required	Container A: V_A = 0.62 m^3, n_A = 1.5 mol, T_A = 300 K. Container B: V_B = 0.21 m^3, n_B = 1.0 mol, T_B = 400 K. R = 8.3 J/(mol·K). Find (a) the final temperature T, and (b) the final pressure p.
Representation	Figure 1-4-4 shows the apparatus — containers A and B joined by a stopcock — and Figure 1-4-5 carries the values: the two containers before opening, and the combined gas of 0.83 m^3 holding 2.5 mol at temperature T after opening.
Strategy	The containers are adiabatic (no heat crosses their walls) and rigid, so the gases simply mix and the total internal energy is conserved: equate the internal energy before and after to find T. Then apply the equation of state $pV = nRT$ to the combined volume and total amount to find p.
Annotated solution	**(a)** Since the gases are simply mixed, the internal energy of the system is conserved. Let the final temperature be T [K]. From conservation of internal energy: $(3/2)\times1.5\times8.3\times300 + (3/2)\times1.0\times8.3\times400 = (3/2)\times(1.5+1.0)\times8.3\times T$ The common factor $(3/2)\times8.3$ cancels: $1.5\times300 + 1.0\times400 = 2.5\times T$, so $850 = 2.5T$ and T = **3.4×10^2 K**. **(b)** Let the final pressure be p [Pa]. From the equation of state $pV = nRT$: $p\times(0.62+0.21) = 2.5\times8.3\times(3.4\times10^2)$ $p = (2.5\times8.3\times340)/0.83 = 7055/0.83$ = **8.5×10^3 Pa**.
Reasonableness check	(a) The result 340 K lies between 300 K and 400 K, nearer 300 K because container A holds more gas (1.5 mol vs 1.0 mol) — exactly what a mole-weighted average gives. (b) With 2.5 mol spread through 0.83 m^3 at 340 K, $p \approx 8.5\times10^3$ Pa, well below atmospheric pressure, which fits a dilute gas in a large volume.
Explain it yourself	In one sentence, say why the final temperature is closer to 300 K than to 400 K, and why the $(3/2)R$ factor can be canceled from both sides of the internal-energy balance.

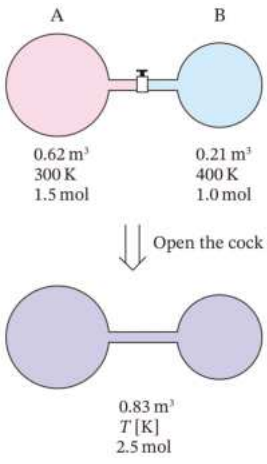

Figure 1-4-5 · Worked Example: the two containers before the stopcock is opened (0.62 m^3, 300 K, 1.5 mol and 0.21 m^3, 400 K, 1.0 mol) and the combined gas afterwards (0.83 m^3, T, 2.5 mol).

✍️ **Now you try:** Adiabatic container A (0.50 m^3) holds 2.0 mol of a monatomic ideal gas at 350 K, and adiabatic container B (0.30 m^3) holds 3.0 mol of the same gas at 300 K. Find the final temperature after the stopcock is opened.

🔆 Try

1. Determine the internal energy in J of a monatomic ideal gas at 27°C and 0.40 mol. Take the gas constant as 8.3 J/(mol·K) and give the answer to 2 significant figures.

2. 0.60 mol of a monatomic ideal gas at 400 K is enclosed in an adiabatic container, and the temperature is changed to 300 K. Determine the change in internal energy in J, to 2 significant figures. Take the gas constant as 8.3 J/(mol·K).
3. An adiabatic container A with a volume of 0.24 m^3 and an adiabatic container B with a volume of 0.40 m^3 are connected by a thin tube with a stopcock. With the stopcock closed, 20 mol of a monatomic ideal gas at 320 K is enclosed in container A, and container B is a vacuum. For the state after the stopcock is opened and a sufficient amount of time has passed, determine to 2 significant figures (i) the temperature of the gas in K, and (ii) the pressure of the gas in Pa. Take the gas constant as 8.3 J/(mol·K).

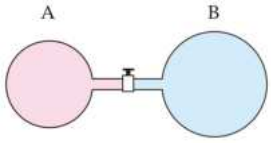

Figure 1-4-6 · Try 2: adiabatic containers A (0.24 m^3) and B (0.40 m^3) joined by a thin tube with a stopcock.

🌐 In a new context

Because the internal energy of a monatomic ideal gas depends only on its temperature and amount, a rigid sealed tank of such a gas acts as a store of thermal energy. If the tank is rigid the gas does no work as it is warmed, so by the first law every joule delivered to it raises the internal energy directly: $\Delta U = (3/2)nR\Delta T$. This is why a fixed-volume gas thermometer responds so predictably — the stored internal energy is tied to the absolute temperature alone.

Figure 1-4-7 · In a constant-volume gas thermometer, heating the sealed rigid bulb makes the molecules move faster and raises the pressure shown on the gauge.

➤ **Your turn:** A rigid, insulated container holds 0.50 mol of a monatomic ideal gas. Its internal energy is raised by 1.5×10^3 J. Using $\Delta U = (3/2)nR\Delta T$ with $R = 8.3$ J/(mol·K), estimate the temperature rise ΔT.

🧪 Nature of Science

From the original — "absolute zero", the ultimate low temperature where thermal motion stops. As the temperature of a gas is lowered, the thermal motion of its molecules becomes calmer and the internal energy decreases. If we extrapolate without limit, we reach a temperature where molecular thermal motion would stop completely. This ultimate low temperature is called "absolute zero" and corresponds to −273.15°C. The absolute (Kelvin) scale takes this point as its reference, 0 K.

Notice that absolute zero is reached by extending a model, not by any measurement: no experiment ever cools matter all the way to it. The relation $U = (3/2)nRT$ points to the same limit, since it predicts the internal energy would fall to zero at $T = 0$. A powerful physical model can locate a boundary that lies beyond every experiment yet still organizes what we can measure.

Your turn: Suggest one reason a real gas cannot actually be cooled to absolute zero, and name one assumption of the ideal-gas model that would break down as that limit is approached.

Lesson question, answered — Two numbers, and no others: how much gas there is, and how hot it is. Everything else you might think matters — which element it is, how heavy each atom is, how big the vessel is, what the pressure reads — drops out of the derivation on the way. That is why the mixing problem could be answered without ever using the two volumes, and why the answer to a temperature-rise question never asks which gas is in the flask. The catch is that this clean result belongs to single atoms in the ideal limit; give the molecules bonds to spin and stretch, or crowd them close enough to feel one another, and the store stops being so simple.

The scene we opened with.

📋 Exercise

1. Determine the internal energy in J of a monatomic ideal gas of temperature T_0 [K] and n [mol], taking the gas constant as R [J/(mol·K)].

2. What is the internal energy in J of a monatomic ideal gas of 27°C and 0.60 mol? Take the gas constant as 8.3 J/(mol·K) and answer to 2 significant figures.
3. A monatomic ideal gas of temperature 100 K and 0.20 mol is enclosed in an adiabatic container, and the temperature is changed to 300 K. What is the change in internal energy in J? Take the gas constant as 8.3 J/(mol·K) and answer to 2 significant figures.
4. An adiabatic container A with a volume of 0.62 m^3 and an adiabatic container B with a volume of 0.21 m^3 are connected by a thin tube with a stopcock. With the stopcock closed, a monatomic ideal gas at 300 K, 14 mol is enclosed in container A, and gas at 400 K, 6.0 mol is enclosed in container B. For the state a long time after the stopcock is opened, answer to 2 significant figures (i) the temperature of the gas in K, and (ii) the pressure of the gas in Pa. Take the gas constant as 8.3 J/(mol·K).
5. An adiabatic container A with a volume of V_1 [m^3] and an adiabatic container B with a volume of V_2 [m^3] are connected by a thin tube with a stopcock, and the stopcock is closed. A monatomic ideal gas of temperature T_1 [K] and n_1 [mol] is enclosed in container A. Taking the gas constant as R [J/(mol·K)], determine the temperature T [K] and pressure p [Pa] a sufficient amount of time after the stopcock is opened, for each case: (i) container B is a vacuum before opening; (ii) a monatomic ideal gas of temperature T_2 [K] and n_2 [mol] is enclosed in container B before opening.

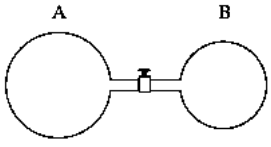

Figure 1-4-8 · Exercise 4: adiabatic containers A (0.62 m^3) and B (0.21 m^3) joined by a thin tube with a stopcock.

1-5 Changes in the State of Gas and Heat/Work

❓ **Lesson question:** How does the state of a gas change when heat is supplied to it or it is compressed?

You will learn to

1. Keep an energy account for a trapped gas, converting freely between quantities referred to the gas and to its surroundings without losing a sign.
2. Read from what a problem holds fixed which entry of that account falls away, and obtain the mechanical entry from the volume a piston sweeps out.
3. Predict which way the temperature moves when a gas is squeezed or released while no heat can cross its walls.

🔍 The phenomenon — predict before you continue

Cap the nozzle of a bicycle pump with a fingertip and push the plunger in hard and fast. No flame touches the air inside, yet the barrel grows noticeably warm near the sealed end. You added no heat, only work — you squeezed the trapped gas — and its temperature rose. Compressing a gas quickly and heating a gas can leave it in the very same warmer state.

★ **Key idea**
Energy reaches a trapped gas as heat Q_{in} [J] or work W_{in} [J]. Which way does each move its store U [J]?

Figure 1-5-1 · A sealed bicycle pump: a fingertip caps the nozzle while the plunger is pushed in, compressing and warming the trapped air.

Predict

(a) When you push the plunger in quickly with the nozzle sealed, does the internal energy of the trapped air rise, fall, or stay the same?

(b) If instead you let the gas expand while no heat can enter or leave, does its temperature rise, fall, or stay the same?

🔍 Explore — Squeeze the trapped air and read the heat

What you'll do: measure how the temperature of a sealed pocket of air changes when you do work on it by compressing it with no heat added, to see work alone raise its internal energy.

Time: 15 min

You will need:

- a bicycle pump
- a fingertip or cap to seal the nozzle
- a stick-on temperature strip
- a ruler to mark a fixed push distance

bicycle pump

temperature strip

ruler

Steps

1. Seal the nozzle with a fingertip, tape the temperature strip to the barrel near the sealed end, and read the starting temperature T_{before}.
2. Push the plunger in quickly through the marked fixed distance so almost no heat escapes (Run 1), then read T_{after} at once.
3. Let the barrel cool back, repeat pushing slowly over several seconds (Run 2); then, with the air held compressed, release the plunger and let the gas expand (Run 3) and read again.
4. For each run work out $\Delta T = T_{after} - T_{before}$ and note its sign in the last column.

Run	Action	T_{before} (°C)	T_{after} (°C)	$\Delta T = T_{after} - T_{before}$ (°C)
1	compress fast			
2	compress slowly			
3	let it expand			

Figure 1-5-2 · The pump with its outlet sealed and a thermometer strip taped near the sealed end; a hand pushes the plunger to do work on the trapped gas.

★ Working note

Read the strip within a second of the push: the warm barrel starts leaking heat to the room at once, so a slow reading hides the rise. Let it cool fully back to room temperature before each new run.

Analyze what you collected:

(a) Look down the last column: in which runs is ΔT positive and in which is it negative? How does the sign of ΔT for the fast compression compare with the sign for the release?

(b) No flame touches the air ($Q_{in} \approx 0$), so what does a positive ΔT tell you about the work done on the gas and its internal energy? Predict the sign of ΔU when the gas instead expands and pushes the plunger out.

✎ Write your explanation in two lines before you read the next page.

From observation to model

Two doors, one account

Energy reaches a trapped gas through exactly two openings and no others. One is thermal: the wall is colder or hotter than the gas, and energy seeps across it without anything moving. The other is mechanical: something shoves the boundary, or the gas shoves it back. Whatever arrives through either opening has nowhere else to go, so it lands in the store the previous lesson measured — which is all the first law

asserts. It is a piece of accounting rather than a discovery about gases, and it would read the same for a spring or a battery.

What makes it feel harder than it is, is that each opening can be described from either side of the wall. A joule the gas absorbs is a joule the flame lost; a joule the gas spends pushing a piston is a joule the piston gained. So every quantity in this lesson comes in a matched pair whose members differ by a minus sign and nothing else. Fix the gas as the system before you write anything down, refer every symbol to it, and treat a negative answer as a direction — energy leaving — rather than as a mistake.

Switch one term off, and a named change appears

Each of the four named changes is the same account with one entry silenced, and the interest lies in *which* entry and *why*. Pin the volume and the boundary never travels; a force that moves nothing transfers nothing, so the mechanical entry is empty and all the heat shows up as a temperature rise. Pin the pressure instead, and the work can be read off a volume change alone: the gas presses its piston of area A with force pA and drives it a distance L, giving

$$W = FL = pA \times L = p \times AL$$

— and AL is simply the volume swept out, so nothing about the piston's shape or size survives into the answer. Pin the temperature, and lesson 1-4 does the work for us: the store of an ideal gas is fixed by amount and temperature alone, so a fixed temperature freezes the store, and heat that enters must leave again as work in the same instant, joule for joule, while pV holds its value. Seal the gas thermally instead, and only the mechanical door is left: everything done on the gas is added to the store, which is why a quick squeeze warms a gas and a quick expansion chills it — the pump barrel and the aerosol can in your hand.

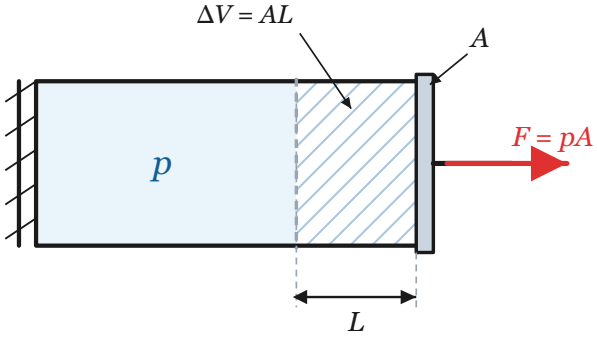

Figure 1-5-3 · At constant pressure p the gas pushes a piston of area A a distance L, doing work $W = FL = pA \times L = p\Delta V$ on it.

★ **Key fact**

The scale of the mechanical door: a piston of area 1.0×10^{-3} m² driven 0.10 m against 1.0×10^{5} Pa carries just 10 J — small beside the heat a burner delivers in a second, which is why gentle expansions matter far less to the energy account than strong heating does.

★ **Term**

How ideal is "adiabatic"? — no wall insulates perfectly, so the label is really a claim about speed: the change must finish before an appreciable amount of heat has time to seep through. The very same stroke of a pump is nearly adiabatic when it is fast and nearly isothermal when it is slow, and every real process sits somewhere between the two idealizations rather than on either.

Point

First Law of Thermodynamics [Review]
The change in internal energy ΔU[J] of a gas is given by:
$\Delta U =$ $Q_{in}+W_{in}$ where Q_{in}[J] is the heat absorbed by the gas and W_{in}[J] is the work done on the gas.
Let the heat released by the gas be Q_{out}, $Q_{out} = -Q_{in}$
Let the work done by the gas be W_{out}, $W_{out} = -W_{in}$

Isochoric change (Change while keeping the volume of the gas constant)
In an isochoric change, $W_{out} =$ 0

Isobaric change (Change while keeping the pressure of the gas constant)
In an isobaric change, $W_{out} =$ $p\Delta V$

Isothermal change (Change while keeping the temperature of the gas constant)
For an isothermal process, since $\Delta U = 0$, $W_{out} =$ Q_{in}

For an isothermal process of an ideal gas, from the equation of state, pV = (constant).

Adiabatic process (Process with no heat transfer)
For an adiabatic process, since $Q_{in} = 0$, W_{in} = ΔU

Equation box	
First law	$\Delta U = Q_{in} + W_{in}$ (with $Q_{out} = -Q_{in}$, $W_{out} = -W_{in}$)
Isochoric	$W_{out} = 0 \implies \Delta U = Q_{in}$
Isobaric	$W_{out} = p\Delta V$ (from $W = FL = pA \times L = p\Delta V$)
Isothermal	$\Delta U = 0 \implies Q_{in} = W_{out}$, pV = constant
Adiabatic	$Q_{in} = 0 \implies \Delta U = W_{in}$

Note
Signs cost more marks here than physics does. Before substituting anything, write down which quantity the problem handed you — heat *into* the gas or out of it, work done *on* the gas or by it — and convert it to the "in" convention first. An expanding gas always gives $W_{in} < 0$, so a compression that raises the temperature and an expansion that lowers it are the same equation read in two directions.

Worked Example

Situation	Determine the following to 2 significant figures. (a) For the following changes, determine the work done on the gas W [J] and the change in internal energy ΔU [J]. (i) 45 J of heat is supplied to a gas at constant volume. (ii) 70 J of heat is supplied to a gas at a constant pressure of 1.0×10^{5} Pa, and its volume expands by 3.0×10^{-4} m^3. (iii) 75 J of heat is supplied to an ideal gas at constant temperature. (b) A gas expands in an adiabatic container. If the work done by the gas is 45 J, determine the change in its internal energy in J.
Given and required	Take the gas as the system. (a)(i) Q_{in} = 45 J, constant volume; find W (= W_{in}) and ΔU. (a)(ii) Q_{in} = 70 J, $p = 1.0\times10^{5}$ Pa, $\Delta V = 3.0\times10^{-4}$ m^3. (a)(iii) Q_{in} = 75 J, constant temperature. (b) $Q_{in} = 0$ (adiabatic), work done by the gas W_{out} = 45 J; find ΔU.
Representation	See Figure 1-5-3 for the isobaric case (ii), where the gas does work $p\Delta V$ on the piston. In each part fix which of Q_{in}, W, and ΔU is set by the type of change before substituting.
Strategy	Check the heat supplied to the gas, the work done by the gas on the surroundings, and the change in internal energy, paying attention to their signs, and substitute them into $Q_{in} = \Delta U + W_{out}$ (equivalently $\Delta U = Q_{in} + W_{in}$). Recall W on the gas is $W_{in} = -W_{out}$.

Annotated solution	**(a)(i)** Since the volume is constant, $W_{out} = 0$, so $W =$ **0 J**; then $\Delta U = Q_{in} + W = 45 + 0 =$ **45 J**. **(a)(ii)** At constant pressure $W_{out} = p\Delta V = (1.0\times10^{5})\times(3.0\times10^{-4}) = 30$ J, so the work done on the gas is $W = -W_{out} =$ **−30 J**; then $\Delta U = 70 - 30 =$ **40 J**. **(a)(iii)** For an isothermal change of an ideal gas $\Delta U = 0$, so $W_{out} = Q_{in} = 75$ J and $W =$ **−75 J**, $\Delta U =$ **0 J**. **(b)** The container is adiabatic, so $Q_{in} = 0$. The work done by the gas is $W_{out} = 45$ J, so the work done on the gas is $W = -45$ J; then $\Delta U = Q_{in} + W = 0 + (-45) =$ **−45 J**.
Reasonableness check	(i) With no work done, all 45 J of heat becomes internal energy — the gas warms. (ii) The gas spends 30 J pushing its boundary out, so only 40 J of the 70 J is left to raise the internal energy. (iii) At constant temperature the internal energy cannot change, so every joule of heat leaves again as work. (b) With no heat in, the expanding gas can only pay for its 45 J of work by losing 45 J of internal energy, so it cools.
Explain it yourself	In one sentence, say why the sign of W (work done on the gas) is negative whenever the gas expands, and why an isothermal change of an ideal gas leaves $\Delta U = 0$.

✍️ **Now you try:** A gas at a constant pressure of 1.0×10^{5} Pa absorbs 60 J of heat, and its volume expands by 2.0×10^{-4} m^3. Find W_{in} and ΔU for this change.

🔆 Try

1. Determine the following to 2 significant figures. For each change, determine the work done on the gas W [J] and the change in internal energy ΔU [J].
   (i) 30 J of heat is supplied to a gas at constant volume.
   (ii) 75 J of heat is supplied to a gas at a constant pressure of 1.0×10^{5} Pa, and its volume expands by 3.0×10^{-4} m^3.
   (iii) 60 J of heat is supplied to an ideal gas at constant temperature.
2. A gas expands in an adiabatic container. If the work done by the gas is 50 J, determine the change in its internal energy in J.

🌐 In a new context

A diesel engine carries no spark plug. On its compression stroke the piston squeezes the air in the cylinder so quickly that almost no heat escapes — an adiabatic compression. All the work done on the air becomes internal energy, $W_{in} = \Delta U$, and the temperature climbs high enough that fuel sprayed into the hot air ignites on its own. The same first law that cools an expanding gas heats a compressed one.

➤ **Your turn:** Air is compressed adiabatically in a cylinder and 80 J of work is done on it. Using $Q_{in} = 0$, find the change in its internal energy ΔU, and say whether the air warms or cools.

Nature of Science

From the original — the "fire piston" that creates fire by compressing air. Put a small amount of cotton in a transparent cylinder fitted with a piston and push the piston in forcefully: the cotton suddenly ignites and burns. Rapidly compressing the gas is an adiabatic compression, so work is done on the gas from the outside, its internal energy rises sharply, and its temperature soars. Diesel engines use the same mechanism, injecting fuel into air heated by adiabatic compression to ignite it.

The fire piston is a clean test of a physical model: the first law predicts that with no heat transfer the work done on the gas must reappear entirely as internal energy, and the flash of the cotton makes that hidden energy visible. A single, repeatable demonstration can turn an abstract equation into something you can see.

That heat and work are two ways of paying the same bill was established by Joule in the 1840s, who churned water with falling weights and measured the temperature rise; the equivalence he found is what the first law states. A law of physics often begins as a stubborn measurement, repeated until no one can dismiss it.

Your turn: Suggest one everyday device or event that heats or cools a gas mainly by doing work on it rather than by adding heat, and identify whether the process is closer to adiabatic or isothermal.

Figure 1-5-4 · At the top of a diesel engine's compression stroke, air squeezed into a small, glowing-hot space ignites the fine cone of fuel sprayed from the injector, with no spark plug needed.

Figure 1-5-5 · A fire piston: driving the piston down fast compresses the trapped air adiabatically, and the temperature soars enough to flash the cotton into flame.

Lesson question, answered — The state of a gas is set by its internal energy, and the first law $\Delta U = Q_{in} + W_{in}$ shows two ways to change it: supplying heat and doing work. Which quantities move depends on what is held fixed — volume ($W_{out} = 0$), pressure ($W_{out} = p\Delta V$), temperature ($\Delta U = 0$), or heat flow (adiabatic, $Q_{in} = 0$) — so compressing a gas can warm it exactly as adding heat would.

The scene we opened with.

Exercise

1. Determine the following to 2 significant figures. For each change, determine the work done on the gas W [J] and the change in internal energy ΔU [J].
   (i) 15 J of heat is supplied to a gas at constant volume.
   (ii) 25 J of heat is supplied to a gas at a constant pressure of 1.0×10^{5} Pa, and its volume expands by 4.5×10^{-4} m^3.
   (iii) 80 J of heat is supplied to an ideal gas at constant temperature.
2. 35 J of heat is supplied to a gas at a constant pressure of 1.0×10^{5} Pa, and its volume expands by 2.0×10^{-4} m^3. Determine the work done on the gas W [J] and the change in internal energy ΔU [J].
3. 75 J of heat is supplied to a gas at constant volume. Determine the work done on the gas W [J] and the change in internal energy ΔU [J].
4. A gas expands in an adiabatic container. If the work done by the gas is 35 J, determine the change in its internal energy in J.

1-6 Molar Heat Capacity

❓ **Lesson question:** How much heat warms one mole of gas by one kelvin, and why does it depend on holding the volume or the pressure constant?

You will learn to

1. Read from a problem which condition is being held while a gas warms, and pick the matching capacity before any number is substituted.
2. Convert between the two capacities of one gas with Mayer's relation, and explain from the energy account why they differ by exactly one gas constant.
3. Quote both values for a single-atom gas as multiples of R, and say what changes once the molecules are no longer single atoms.

🔍 The phenomenon — predict before you continue

Put the same amount of heat under two pots. A sealed pressure cooker traps its steam, so the pressure climbs and the water inside can pass 100 °C before it boils; an open pot stays at atmospheric pressure, so its water sticks at 100 °C and the surplus heat leaves as steam. That everyday difference is about boiling — but it turns on the distinction this lesson is built from. Hold a gas at constant volume and every joule supplied drives its temperature up; let it expand at constant pressure instead and part of that heat is spent pushing the surroundings back. Give both the same heat, and one ends up hotter than the other.

★ **Key idea**

Molar heat capacity [J/(mol·K)] — the heat 1 mol needs per 1 K. Do C_V (fixed V) and C_p (fixed p) agree?

Figure 1-6-1 · The same heat under a sealed pressure cooker (constant volume) and an open pot (constant pressure). In the kitchen the sealed pot runs hotter because trapped steam raises the boiling point; for a gas, because it spends none of the heat on expanding.

Predict

(a) After the same heat is supplied, which sample ends up hotter — the one at constant volume or the one at constant pressure?

(b) In which sample does the gas do work on its surroundings as it is heated?

🔍 Explore — Same heat step, two containers

What you'll do: Warm the same gas by the same temperature step in a fixed-volume flask and in a free-piston cylinder, and measure how much extra heat the constant-pressure setup demands.

Time: 15 min

You will need:

- A rigid flask (constant volume)
- A cylinder closed by a freely sliding piston (constant pressure)
- A joule-counting heater to supply the same gas sample to each
- A thermometer for each container
- The amount of gas n [mol] sealed in each container, stated on the apparatus

Rigid flask

Piston cylinder

Heater

Thermometer

Steps

1. Put the same amount n of the same gas in the rigid flask and in the piston cylinder at the same starting temperature.
2. Warm each with the identical heater and read the heat supplied until its thermometer rises by the chosen ΔT; record Q_V (flask) and Q_p (cylinder).
3. Compute the derived column $(Q_p - Q_V) / (n\Delta T)$ — the extra heat per mole per kelvin the constant-pressure setup needed.
4. Repeat for a different ΔT (and, if you can, a different n) and compare the derived column across runs.

Run	Q_V at constant V (J)	Q_p at constant p (J)	$(Q_p - Q_V) / (n\Delta T)$ [J/(mol·K)]
1			
2			
3			

Analyze what you collected:

(a) In every run, which container needed more heat for the same ΔT, and does the derived column $(Q_p - Q_V) / (n\Delta T)$ stay the same as you change ΔT?

(b) That steady value equals one well-known constant — which one? Use it to predict $C_p - C_V$ for any ideal gas before you meet Mayer's relation.

✎ Write your explanation in two lines before you read the next page.

Figure 1-6-2 · The same gas in a rigid flask (constant V) and in a piston cylinder (constant p), each with a thermometer and warmed by an identical heater.

★ **Working note**

Use the same n and the same target ΔT for both containers each run, and let the piston move freely so its pressure stays truly constant while it rises.

From observation to model

Why the question is incomplete until you say what is held fixed

Ask how much energy raises a gas by one degree and you have not yet asked a well-posed question: it depends on what the gas is permitted to do while it warms. A metal block has nowhere to go, so a single number describes it. A gas can shove its boundary outwards, and if it does, part of the energy you supply leaves again through the mechanical door of the previous lesson before the thermometer ever registers it. Two careful experiments on the very same gas therefore return two

different numbers — and neither is wrong. Physics keeps both, and tags each with the condition that produced it. Quoting such a number for a gas without naming that condition is like quoting a length without a unit.

Deriving the gap between them

Run the two experiments side by side on n mol raised by ΔT. With the volume locked, the boundary never travels, so the energy account of lesson 1-5 has a single entry and every joule supplied ends up in the store: the heat needed is just the rise in internal energy. With the pressure locked instead, that same rise in the store is still required — for an ideal gas the store answers to temperature alone, so it cannot care which experiment you are running — but the gas now also drives its piston and pays out $p\Delta V$ on the way. Subtract one experiment from the other and the difference in the heat supplied is exactly that payment. The equation of state then prices it: at fixed pressure, $pV = nRT$ forces

$$p\Delta V = nR\Delta T$$

so the extra heat per mole per kelvin is R itself — no property of the particular gas survives the division, which is why the relation is quoted for every ideal gas at once.

The two monatomic values come from the same place. Lesson 1-4 fixed the store of a single-atom ideal gas at $(3/2)nRT$, so raising it by ΔT costs $(3/2)nR\Delta T$; at locked volume that is the entire bill, and dividing by $n\Delta T$ leaves one and a half gas constants. Adding the piston's share from the relation just derived raises it by one more, giving two and a half. Both are pure multiples of R with no trace of helium or argon left in them — lesson 1-4's result about the store, reappearing here wearing a different name.

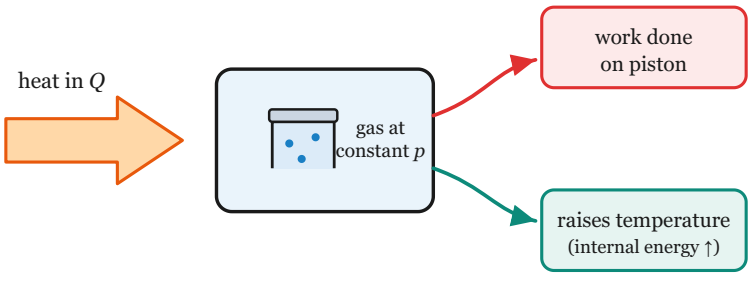

Figure 1-6-3 · At constant pressure the incoming heat Q splits: part raises the internal energy (temperature), part does work on the rising piston.

Point

- The amount of heat required to change the temperature of 1 mol of gas by 1 K is called its molar heat capacity. In an isochoric process, it is called the **isochoric molar heat capacity C_V** [J/(mol·K)], and in an isobaric process, it is called the **isobaric molar heat capacity C_p** [J/(mol·K)].
  Therefore, the amount of heat Q[J] required to change the temperature of n[mol] of gas by ΔT[K] is:
  In an isochoric process, $Q =$ **$nC_V\Delta T$**, and in an isobaric process, $Q =$ **$nC_p\Delta T$**.
- The isochoric molar heat capacity and the isobaric molar heat capacity are related by **$C_p = C_V + R$** (Mayer's relation).
- The molar heat capacity at constant volume C_V[J/(mol·K)] and the molar heat capacity at constant pressure C_p[J/(mol·K)] for a monatomic ideal gas can be expressed in terms of the gas constant R as $C_V =$ **$(\frac{3}{2})R$** and $C_p =$ **$(\frac{5}{2})R$**.

★ **Key fact**

The size of these numbers: with R = 8.3 J/(mol·K), a single-atom gas comes out near 12 J/(mol·K) held at fixed volume and near 21 J/(mol·K) held at fixed pressure. Warming one mole of helium by one kelvin against the atmosphere therefore costs about as much energy as lifting a 2 kg book one meter.

★ **Term**

How far the two numbers travel — $(3/2)R$ and $(5/2)R$ are the single-atom case and nothing more. A two-atom gas can also spin, so it stores more per kelvin and its constant-volume value climbs toward $(5/2)R$; hotter still, the bond begins to vibrate and it climbs again. Mayer's relation itself survives all of this, because its derivation uses only the equation of state — but the two monatomic values do not.

★ **Note**

The routine slip is picking the wrong capacity. Before substituting, hunt the problem for the words that fix the condition — "rigid", "sealed", "fixed volume" on one side; "free piston", "open to the atmosphere", "constant pressure" on the other — and only then choose C_V or C_p for $Q = nC\Delta T$. When a problem hands you one capacity and asks for the other, convert with Mayer's relation instead of starting the calculation again.

📊 Equation box	
Heat at constant volume	$Q = nC_V\Delta T$ (C_V in J/(mol·K))
Heat at constant pressure	$Q = nC_p\Delta T$ (C_p in J/(mol·K))
Mayer's relation	$C_p = C_V + R$
Monatomic ideal gas	$C_V = (\frac{3}{2})R$ $C_p = (\frac{5}{2})R$

🛠 Worked Example

Situation	If 170 J of heat is supplied to 0.80 mol of an ideal gas at constant volume, its temperature rises by 10 K. Determine the isochoric molar heat capacity of this gas in J/(mol·K) to 2 significant figures.
Given and required	Q = 170 J, n = 0.80 mol, constant volume (isochoric), ΔT = 10 K; find C_V in J/(mol·K).
Representation	A fixed amount of gas in a rigid container that cannot change volume, warmed by a supplied heat Q (see Figure 1-6-2, rigid flask).
Strategy	Decide whether the process is isochoric or isobaric, then set up $Q = nC\Delta T$ using the matching molar heat capacity. Here the volume is constant, so use $Q = nC_V\Delta T$ and solve for C_V.
Annotated solution	In an isochoric process, the heat to change the temperature of 1 mol by 1 K is C_V, so $Q = nC_V\Delta T$. Thus 170 = 0.80 × C_V × 10. C_V = 170 / (0.80 × 10) = 170 / 8.0 = 21.25 ≈ **21 J/(mol·K)**.
Reasonableness check	The result, about 21 J/(mol·K), is a few times the gas constant R ≈ 8.3 J/(mol·K) — the right order of magnitude for a molar heat capacity — so an answer near 21 J/(mol·K) is sensible.
🧐 **Explain it yourself**	In one sentence, say why choosing C_V rather than C_p is the correct step when the container is rigid.

✍️ **Now you try:** If 250 J of heat is supplied to 0.50 mol of an ideal gas at constant volume, its temperature rises by 20 K. Find its isochoric molar heat capacity to 2 significant figures.

🔆 Try

1. If 63 J of heat is supplied to 1.5 mol of an ideal gas at constant pressure, its temperature rises by 2.0 K. Determine the isobaric molar heat capacity C_p and the isochoric molar heat capacity C_V of this gas in J/(mol·K), respectively, to 2 significant figures. Let the gas constant be 8.3 J/(mol·K).

🌐 In a new context

The ratio $\gamma = C_p/C_V$ reaches far beyond a heating experiment: it sets the **speed of sound** in a gas. A sound wave squeezes and stretches the air far too quickly for heat to leak away, so the springiness the wave feels is governed by γ, and the speed works out as $\sqrt{\gamma RT/M}$, where M is the molar mass in kilograms per mole. Timing how fast sound crosses a room therefore measures γ for air — and because γ is about 1.67 for a single-atom gas but about 1.4 for a two-atom gas, that simple timing reveals that air is built from molecules of two atoms, not lone atoms.

➤ **Your turn:** Using R = 8.3 J/(mol·K), find C_V and C_p for a monatomic ideal gas to 2 significant figures.

Figure 1-6-4 · Two identical gas samples are heated equally: the locked left piston keeps the volume constant, while the free right piston rises above its original level as the gas expands at constant pressure.

🧪 Nature of Science

From the original — why it is harder to raise the temperature in an isobaric process. The heat to raise 1 mol of gas by 1 K is the molar heat capacity, but more heat is needed at constant pressure than at constant volume. When heated at constant pressure the gas expands and does work on the outside by pushing up the piston. Since part of the supplied thermal energy is used for that work, extra heat must be supplied to raise the temperature by the same amount.

This is a case of energy accounting: physicists separated the incoming heat into a part that raises the internal energy and a part that leaves as work. That split is not something you can see directly — it is inferred by demanding that energy be conserved. Mayer's relation $C_p = C_V + R$ is the quantitative statement of that inference, and it is confirmed whenever the measured difference of the two molar heat capacities comes out equal to R.

Your turn: Suggest one measurement you could make on a real gas to test whether $C_p - C_V$ really equals R, and say which quantities you would need to hold constant.

Figure 1-6-5 · Heated at constant pressure, the gas expands and pushes the piston up from its starting level, doing work on the surroundings.

Lesson question, answered — There is no single answer, and that is the answer: a gas has two heat capacities, one for each way you may hold it while it warms, and the question is only complete once you say which. Locked in a rigid vessel, everything you supply is banked as internal energy. Left free to push a piston, the gas spends part of the supply on the surroundings before the thermometer moves, so more must be supplied for the same rise — and the equation of state prices that spending at exactly one gas constant per mole per kelvin, whatever the gas. For single atoms the two numbers come out as one and a half and two and a half gas constants, straight from the store measured in lesson 1-4; give the molecules a way to spin and both climb.

The scene we opened with.

📋 Exercise

1. Determine the following to 2 significant figures, assuming the gas constant is 8.3 J/(mol·K). If 240 J of heat is supplied to 1.2 mol of an ideal gas at constant volume, its temperature rises by 12 K. Determine the isobaric molar heat capacity C_p of this gas in J/(mol·K).
2. Assuming the gas constant is 8.3 J/(mol·K): if 200 J of heat is supplied to 0.30 mol of an ideal gas at constant pressure, its temperature rises by 25 K. Determine the isobaric molar heat capacity C_p and the isochoric molar heat capacity C_V of this gas in J/(mol·K), respectively.

UNIT 2

Static Electricity

2-1	Coulomb's Law	46
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2-1 Coulomb's Law

❓ **Lesson question:** Rubbing a balloon makes it attract paper scraps or push another balloon away — what decides how strong that push or pull between two charged objects is?

You will learn to

1. Define electric charge and its unit the coulomb (C), and describe how like charges repel and unlike charges attract through the electrostatic force.
2. State Coulomb's law, $F = k|q_1|\cdot|q_2|/r^2$, relating the force between two point charges to the magnitude of each charge and to their separation.
3. Apply Coulomb's law together with the balance of forces on a suspended charged ball to find an unknown electrostatic force or charge.

🔍 The phenomenon — predict before you continue

Rub an inflated balloon on your hair and hold it near small scraps of paper: the scraps leap up and cling to it, even without contact. Rub two balloons the same way, hang each from a thread, and bring them close: now they swing apart instead. The same act of rubbing produces both an attraction and a repulsion, depending on what the balloon meets.

★ **Key idea**
Charge q [C] — of two kinds, + and −. Which pairs push, which pull, and how does F fade with r?

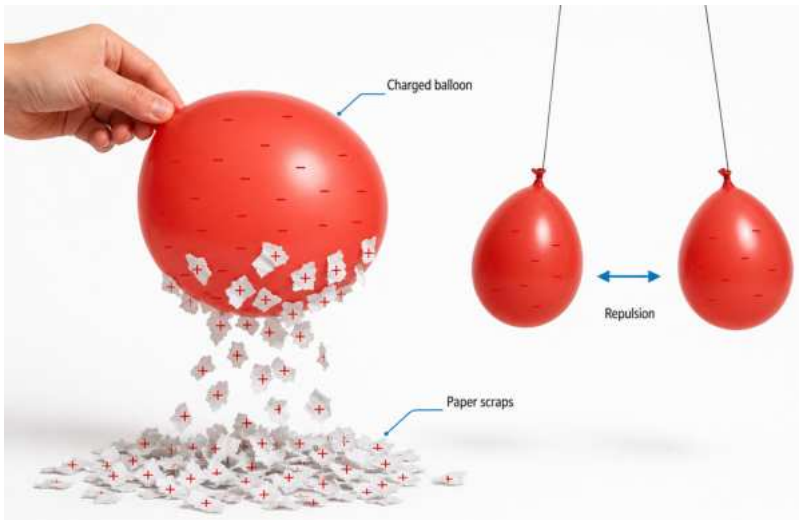


Figure 2-1-1 · A rubbed balloon lifts paper scraps (unlike induced charges attract), while two like-charged balloons on threads swing apart (like charges repel).

Predict

(a) When the balloon attracts the paper, are the balloon's charge and the paper's induced charge alike or unlike?

(b) When the two rubbed balloons swing apart, do they carry like or unlike charges?

🔍 Explore — How the push fades with distance

What you'll do: hang two equally charged foam balls, measure their separation and thread angle at several spacings, and test how the repulsion depends on distance.

Time: 15 min

You will need:

- two light foam balls on insulating threads
- a ruler and a protractor
- a balance to find each ball's mass m
- a charged rod
- a woolen cloth

foam balls on threads

ruler

charged rod

Steps

1. Rub both foam balls equally with the woolen cloth so they carry like charge, then hang them side by side from insulating threads on a common support.
2. Let them settle apart, and with the ruler read the separation r between their centers and the angle θ each thread makes with the vertical.
3. Change the thread length or the spacing of the support points, recharge the balls equally, and record r and θ again.
4. For each run work out the repulsion $F = mg \tan\theta$, then the product $F \times r^2$, and enter both in the table.

Figure 2-1-2 · Two like-charged foam balls on insulating threads hang from a rod and settle apart; a ruler reads their separation r.

Run	Separation r (m)	Angle θ (°)	Force $F = mg \tan\theta$ (N)	$F \times r^2$ (N·m²)
1				
2				
3				

> ★ **Working note**
> Let the balls settle before reading, measure r center-to-center, and recharge both balls equally each run because static charge leaks away quickly in humid air.

Analyze what you collected:

(a) Look down the last column: as r changes from run to run, does $F \times r^2$ stay roughly the same or drift? What does a nearly constant value tell you about how F depends on r?

(b) Predict what happens to the repulsion if you could halve the separation r while keeping the charge on each ball the same, and connect your answer to the idea of a point charge.

✎ Write your explanation in two lines before you read the next page.

From observation to model

Why rubbing needs a quantity of its own

Rubbing does not conjure a pull out of nothing. It transfers something countable from one body to the other, and the transfer can be larger or smaller — which is exactly what a balloon that has been rubbed twice as long demonstrates. That "how much" cannot be reduced to mass, to length, or to time; it behaves as a quantity of its own kind, and it therefore needs a unit of its own before any law can be written down at all. Its sign is not an afterthought either: two possibilities, and only two, are needed to account for both outcomes seen at the start of this lesson — one balloon lifting paper, two rubbed balloons flying apart.

Reading the inverse square out of the table

Your Explore data settle the shape of the law before any formula is written. Each thread angle is first converted into a force by $F = mg \tan\theta$; the last column then does the deciding. If $F \times r^2$ keeps very nearly the same value while r is changed from run to run, no other dependence on distance is left open — the force must fall as $1/r^2$. A second experiment settles the other half: charge one of the balls twice as strongly and the pull doubles, so the two charges enter the relation multiplied together rather than added. Putting the two findings side by side fixes everything except one number, the constant k, whose value has to be measured and cannot be reasoned out.

Turning a deflection into a force

A hanging ball never reports a force directly; it reports an angle. Three pulls hold it still, and only two of them are known in advance — the weight, and the direction (though not the size) of the string tension. Resolving along and across the vertical splits the single equilibrium into two equations, and eliminating the unknown tension between them leaves

$$F = mg \tan\theta$$

The geometry has traded something invisible for something you can read off a protractor (Figure 2-1-3). Once F is known this way, the relation is run backwards — from a measured force to an unknown charge, which is how the Worked Example finds the charge on B.

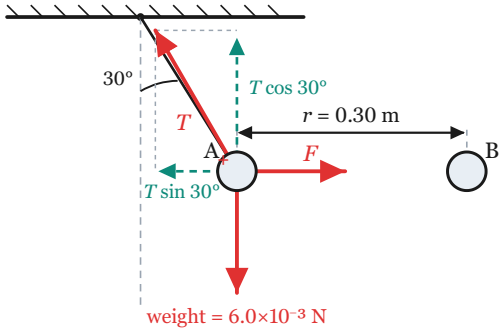


Figure 2-1-3 · Free-body diagram of ball A: the string deflects 30° from the vertical toward B (0.30 m away), so the tension T (with components $T\cos 30°$ up and $T\sin 30°$ across), the electrostatic force F toward B, and the weight 6.0×10^{-3} N balance.

Point

- When an object has electricity, it is said to be electrified, and an electrified object is called a charged body. The electricity that a charged body has is called a charge, and the amount of charge is called the magnitude of the charge. The unit of charge is the **coulomb** (symbol **C**). The positive and negative charges are expressed by + and − signs.
- Like charges repel and unlike charges attract each other. This force is called **electrostatic force**.
- A point-like charge whose size is small enough to be ignored is called a **point charge**.
- The magnitude F [N] of the electrostatic force acting between two point charges is proportional to the magnitude of each electric charge $|q_1|$ [C], $|q_2|$ [C], and inversely proportional to the square of the distance r [m] between the point charges. It is expressed as **$F = k|q_1|\cdot|q_2|/r^2$.** This is called **Coulomb's law**, and k is called the proportionality constant of Coulomb's law.

★ **Key fact**

The coulomb is a huge unit on the laboratory scale. With $k = 9.0\times10^9$ N·m²/C², two charges of 1 C held 1 m apart would push each other with 9.0×10^9 N — so the millinewton forces measured on hanging balls tell you at once that the charges involved are a minute fraction of a coulomb.

★ **Term**

Where the model stops — the law is written for *point* charges, and it is trustworthy only while the separation is large compared with the size of the bodies. Bring two charged spheres almost into contact and each one redistributes the other's charge over its surface; the single distance r then no longer describes the arrangement, and the measured force departs from the prediction.

★ **Note**

Substitute magnitudes only, never signed values. Let the diagram carry the direction — arrows pushing apart for like charges, pulling together for unlike ones — and the arithmetic will not hand you a spurious minus sign.

Equation box	
Coulomb's law	$F = \dfrac{k\lvert q_1\rvert\cdot\lvert q_2\rvert}{r^2}$ (unit: N; $k = 9.0\times10^9$ N·m²/C²)
Equilibrium of a hung ball	Horizontal: $F = T\sin\theta$ Vertical: $T\cos\theta = mg \implies F = mg\tan\theta$

🛠 Worked Example

Situation	A charge of 2.0×10^{-7} C is given to a small ball A suspended by a light string, and a charged small ball B is brought close to it. The two come to the same height and become stationary at a distance of 0.30 m, with the string deflected 30° from the vertical. The gravity acting on A is 6.0×10^{-3} N, the proportionality constant of Coulomb's law is 9.0×10^9 N·m²/C², and $\sqrt{3} = 1.7$. (a) Determine the electrostatic force F acting between A and B. (b) Determine the charge q of B.
Given and required	$q_A = 2.0\times10^{-7}$ C, weight of A = 6.0×10^{-3} N, $r = 0.30$ m, $\theta = 30°$, $k = 9.0\times10^9$ N·m²/C², $\sqrt{3} = 1.7$. Find (a) F, then (b) q of B.
Representation	Figure 2-1-4 sets out the arrangement, and Figures 2-1-3 and 2-1-5 resolve it: ball A hangs on the string and is pushed to 30° from the vertical. The three forces on A are the electrostatic force F (horizontal, toward B), the weight 6.0×10^{-3} N (down), and the string tension T (along the string).
Strategy	Check the distance between the two point charges and their charges, and substitute into Coulomb's law to find the force. First find F from the equilibrium of A, then use Coulomb's law to find the unknown charge. For the direction of the force, draw a repulsive force for like charges and an attractive force for opposite charges.

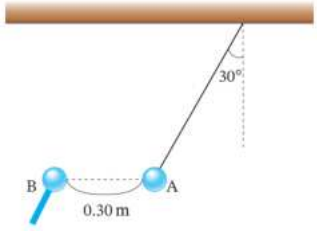


Figure 2-1-4 · Worked Example: ball A hangs from a light string deflected 30° from the vertical, while the charged ball B rests at the same height 0.30 m away.

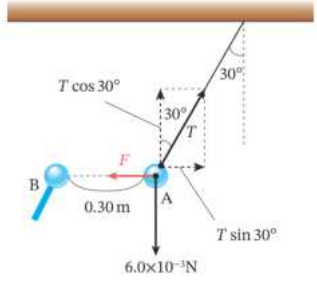


Figure 2-1-5 · Worked Example: the forces on A resolved — the tension T with its components $T\cos 30°$ and $T\sin 30°$, the electrostatic force F, and the weight 6.0×10^{-3} N.

Annotated solution	**(a)** For ball A the electrostatic force F [N], the weight, and the string tension T [N] are in equilibrium. Horizontal: $F - T\sin 30° = 0$ .....(i) Vertical: $T\cos 30° - 6.0\times10^{-3} = 0$ .....(ii) From (i), $\sin 30° = 1/2$, so $T = 2F$. Substituting into (ii): $2F\cos 30° = 6.0\times10^{-3}$, that is $\sqrt{3}\,F = 6.0\times10^{-3}$. $F = 6.0\times10^{-3}/\sqrt{3} = (6.0\times10^{-3}\times 1.7)/3 =$ **3.4×10^{-3} N**. **(b)** By Coulomb's law, $F = k\,q_A\lvert q\rvert/r^2 = 9.0\times10^9\times(2.0\times10^{-7})\times\lvert q\rvert/0.30^2$. The coefficient is $(9.0\times10^9\times 2.0\times10^{-7})/0.090 = 2.0\times10^4$, so $3.4\times10^{-3} = 2.0\times10^4\times\lvert q\rvert$, giving $\lvert q\rvert = 1.7\times10^{-7}$ C. Since an attractive force acts between A and B, their charges have opposite signs. Therefore $q =$ **-1.7×10^{-7} C**.
Reasonableness check	(a) The horizontal pull equals the weight scaled by $\tan 30° \approx 0.58$, so a force a little below 6.0×10^{-3} N is sensible: 3.4×10^{-3} N fits. (b) A charge of order 10^{-7} C on B, comparable to the 2.0×10^{-7} C on A, is consistent with a millinewton force at 0.30 m.
Explain it yourself	In one sentence, say why the deflection of the string lets you find the electrostatic force before you know either charge on B.

✍️ **Now you try:** A charge of 3.0×10^{-7} C is given to a ball A on a light string; a charged ball B is brought to the same height 0.30 m away and the string settles 30° from the vertical. The gravity on A is 6.0×10^{-3} N and $\sqrt{3} = 1.7$. Find F and the charge q of B.

🔆 Try

1. Two small balls A and B, each with a mass of 0.50 kg, are suspended by two light strings of length 0.30 m from a common point. When an equal amount of positive charge is given to A and B, they become stationary with each string making 45° with the vertical. Take the proportionality constant of Coulomb's law as 9.0×10^9 N·m²/C², the gravitational acceleration as 10 m/s², and $\sqrt{2} = 1.4$.
   (a) Determine the electrostatic force F acting between A and B.
   (b) Determine the charge q of A.

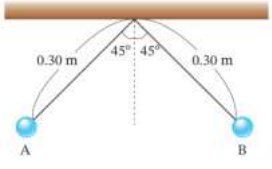


Figure 2-1-6 · Try 1: two balls of mass 0.50 kg hang from a common point on strings of length 0.30 m and swing apart when both are charged alike.

🌐 In a new context

Because Coulomb's law falls off with the square of the separation, small changes in distance change the force sharply. Two charged spheres held at some separation feel a force F; halve the distance and the force does not double but quadruples, since $1/r^2$ grows by a factor of four. This steep dependence is why a charged rod barely disturbs a distant scrap of paper yet snatches a near one, and why keeping like charges apart demands so much less effort as the gap widens.

➤ **Your turn:** Two point charges feel a force of 0.10 N when they are 0.20 m apart. Using $F = k|q_1|\cdot|q_2|/r^2$, estimate the force when the same charges are moved to 0.10 m apart.

Figure 2-1-7 · A charged rod lifts only the paper scraps directly beneath it while scraps a short distance away stay flat and undisturbed, showing how steeply the electrostatic attraction weakens with distance.

Nature of Science

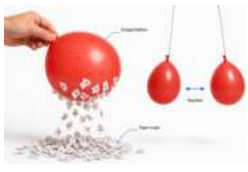
From the original — Column: photocopiers and static electricity. The way letters and images are printed on paper in a photocopier uses the electrostatic force. An invisible image pattern is created by laser light using static electricity on a drum called a photoreceptor, and toner (powder ink) with an opposite charge is attracted and attached to it. The copy is completed by moving it onto paper and fixing it with heat.

Notice what the law leaves out. It says nothing about what the bodies are made of, nothing about the air between them, and nothing about how long ago they were rubbed — an idealization that strips away everything the experimenter can vary without changing the result. Physics advances by discovering which details may be discarded, and the test of such a stripping-down is not that it is literally true of any real ball, but that it keeps predicting correctly and fails in a known, statable direction.

Coulomb measured this law in 1785 with a torsion balance so delicate that the twist of a fiber served as his force meter. Cavendish had reached the same inverse square earlier but did not publish, so the law carries Coulomb's name. What is published, and can be repeated, is what enters science.

Your turn: Suggest one situation where two charged objects could *not* be treated as point charges, and say which assumption breaks down.

Lesson question, answered — Rubbing transfers a measurable quantity, and its two possible signs are enough to explain both a balloon that lifts paper and two balloons that fly apart. The size of the resulting push or pull is set by how much has been transferred to each body and, far more sharply, by how far apart they are: halving the gap multiplies the force fourfold. That single relation is enough to work in either direction — from known charges to a predicted force, or from a hanging ball's deflection back to a charge nobody measured directly.



The scene we opened with.

Exercise

1. Take the proportionality constant of Coulomb's law as 9.0×10^{9} N·m²/C², the gravitational acceleration as 10 m/s², $\sqrt{2}$ = 1.4, and $\sqrt{3}$ = 1.7. A charge of 2.0×10^{-7} C is given to a small ball A suspended by a light string, and a charged small ball B is brought close to it. The two come to the same height and become stationary at a distance of 0.20 m, with the string deflected 60° from the vertical. The gravity acting on A is 3.0×10^{-3} N.
   (i) Determine the electrostatic force F acting between A and B.
   (ii) Determine the charge q of B.

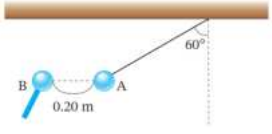


Figure 2-1-8 · Exercise 1: a charged ball A on a light string is deflected by a second charged ball B brought near it.

2. A charge was given to a small ball A with a mass of 2.0×10^{-3} kg suspended by a light string, and a small ball B with a charge of 4.9×10^{-7} C was brought close to it. The two came to the same height and became stationary at a distance of 5.0×10^{-2} m as shown in the figure on the right.
   (i) Determine the electrostatic force F acting between A and B.
   (ii) Determine the charge q of A.
3. Two small balls A and B, each with a mass of 0.25 kg, are suspended by two light strings of length 0.10 m from a common point. When an equal amount of positive charge is given to A and B, they become stationary with each string making 45° with the vertical. Determine the charge of A.

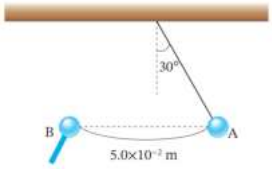


Figure 2-1-9 · Exercise 2: ball A of mass 2.0×10^{-3} kg hangs beside ball B carrying 4.9×10^{-7} C, both at the same height.

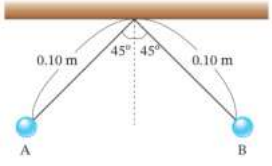


Figure 2-1-10 · Exercise 3: two balls of mass 0.25 kg on strings of length 0.10 m from a common point, charged alike.

2-2 Electric Field

❓ **Lesson question:** Why do forces act between electric charges that are separated from each other?

You will learn to

1. Define the electric field E as the force per unit positive charge, and use $F = qE$ to find the electrostatic force on a charge q placed in a field.
2. Calculate the field of a point charge from Coulomb's law, $E = k|Q|/r^2$, and state its direction — radially outward when Q is positive and radially inward when Q is negative.
3. Find the resultant field of several point charges by adding their fields as vectors, using the superposition principle.

🔍 The phenomenon — predict before you continue

Watch a child near the end of a long plastic slide: the hair streams outward and stands apart while she is still moving. Rubbing along the slide has left her body charged, and that charge fills the space around it with an **electric field**. Each hair carries some of that same charge, so each one feels a force pushing it outward along the field — no hand and no contact required. It is the field, not the body itself, that acts on whatever charge sits in it.

★ **Key idea**

Electric field E [N/C] — what a charge leaves in the space around it. How big is the force on a charge q placed there, and which way does it point?

★ **Term**

Electric field — the book-keeping device this lesson builds. Rather than asking what one charge does to another across a gap, ask what condition the first charge sets up at each point around it; whatever is put there afterwards responds only to the condition where it happens to sit.

Figure 2-2-1 · Near the end of a long plastic slide: rubbing along the way has charged the child, and that charge fills the surrounding space with an electric field — every charged hair feels a force pushing it outward.

Predict

(a) A tiny positive test charge is placed in the field around the charged child. Does the force on it point along the field or opposite to it?

(b) If you double the size of that test charge, what happens to the force it feels?

🔍 Explore — How a charged rod's field weakens with distance

What you'll do: Use a light test charge on a thread to measure how the sideways pull of a charged rod fades as you move away, revealing that the field falls off as $1/r^2$.

Time: 15 min

You will need:

- A plastic rod and a cloth to charge it
- A foil-covered ball hung on a fine thread (the test charge)
- A stand or clamp to hold the rod upright
- A ruler to measure distance and sideways swing

Charged rod

Test ball on thread

Ruler

Steps

1. Rub only the top few centimeters of the plastic rod with the cloth. Charge cannot move along an insulator, so it stays concentrated near the tip and acts as a single point charge. Clamp the rod upright.
2. Touch the foil-covered ball briefly against the charged tip, then let it hang free: only the spot touched gives up charge — none flows to it along the insulator — so the ball leaves with a small charge of the same sign. Because the two charges are alike, note that the thread swings away from the rod.
3. Set the test charge level with the charged tip, at a measured horizontal distance r from the rod, read the sideways swing d of the thread with the ruler, then compute $d \times r^2$ for that run.
4. Recharge the tip and repeat at two larger distances, keeping the ball and thread the same.

Run	Distance r (cm)	Sideways swing d (cm) — a measure of the force	$d \times r^2$
1			
2			
3			

Analyze what you collected:

(a) As r increases, what happens to the swing d? Compare your three values of $d \times r^2$ — do they stay roughly the same?

(b) The swing d measures the force, and the force is proportional to the field. If $d \times r^2$ is nearly constant, what does that tell you about how the field depends on r — and how does it match $E = k|Q|/r^2$?

✎ Write your explanation in two lines before you read the next page.

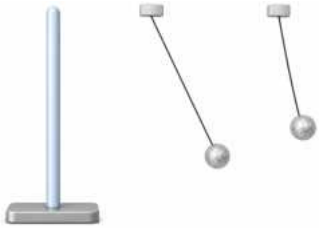


Figure 2-2-2 · A foil-covered ball on a thread, charged by touching the rod, serves as a small test charge of the same sign. Near the charged rod the thread swings far (a strong force); farther away it swings less — tracing the direction and strength of the field.

★ **Working note**

Recharge the tip before each run so its charge Q stays the same, and measure the sideways swing d when the thread has settled and stopped moving. The swing stands for the force only while it stays small next to the thread's length, so keep every run inside that range.

From observation to model

Why an intermediary is worth inventing

Coulomb's law of the previous lesson is silent about the gap. It relates a force to two charges and a separation, as though each body reached across empty space and took hold of the other instantly. Physics is uneasy with that. The alternative is to split the one statement into two: the first charge does something to the space around it, and the second charge then reacts to whatever it finds where it sits. Nothing has to travel between the two bodies for the second step to work — it only has to know its own surroundings.

Dividing out the test charge

The split only earns its keep if the first step can be described without mentioning the second body at all — and it can. Put a small charge q at a chosen point, measure the force on it, then repeat with twice that charge: the force doubles, but the ratio F/q comes out the same. That invariance is the whole argument. What survives dividing out the test charge belongs to the point, not to whatever was placed there to probe it, so F/q can be tabulated once and used for any charge afterwards. It has to be a vector because the force it stands for is one; and since a negative q reverses the force but not the ratio's meaning, the sign of the charge alone flips which way the push goes (Figure 2-2-4).

> ★ **Key fact**
> Newtons per coulomb (N/C) is the unit that falls out of dividing a force by a charge. In the next lesson but one you will meet the same quantity written V/m; the two are the same unit under different names.

From one charge to many

Applying the same division to Coulomb's law is now purely mechanical: divide both sides of $F = k|Q||q|/r^2$ by the probe charge q and the probe disappears from the right-hand side, leaving a quantity that depends only on the source Q and on how far away you are. Its direction is inherited from the force on a positive probe, which is pushed away from a positive source and drawn toward a negative one (Figure 2-2-3).

Several sources are handled just as cheaply. Forces on the probe add as vectors — that is ordinary mechanics, not a new law — and dividing every term of that vector sum by the same q leaves the separate contributions adding in exactly the same way. This is why the parallelogram construction in the Worked Example is legitimate, and why fields of equal size can cancel completely in one direction while reinforcing in another.

> ★ **Note**
> The probe must be small enough not to disturb what it measures. A large probe charge pushes the source's own charges around, and the reading you get then describes an arrangement you created rather than the one you meant to survey.

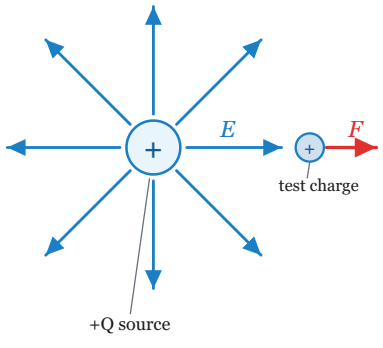


Figure 2-2-3 · The field of a positive point charge points radially outward. A unit positive test charge on a field line feels a force F in the same direction as the field E.

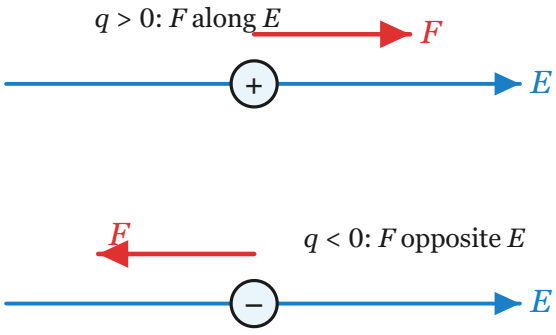
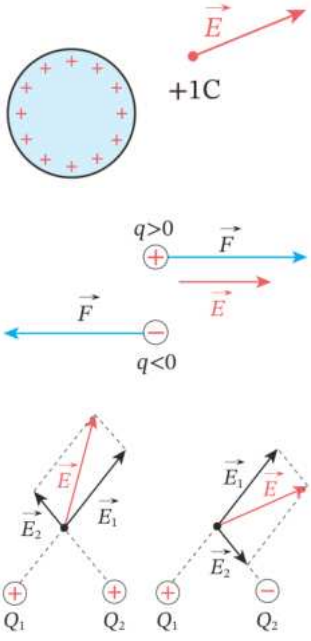


Figure 2-2-4 · In the same field E, a positive charge feels a force F along the field, while a negative charge feels a force along the same line but in the opposite direction.

Point

- The property of space that exerts a force on an electric charge is called an **electric field** . An electric field is a vector quantity with magnitude and direction.
- The electric field E at each point in space is determined by the force that would be exerted on a unit positive charge assumed to be placed at that point.
  Magnitude of electric field E ... magnitude of the force acting on a unit positive charge
  Direction of electric field E ... direction of the force acting on a unit positive charge
  The magnitude of the electrostatic force F [N] acting on a charge q [C] placed at a point with electric field strength E [N/C] is : **$F = qE$**
- A positive charge experiences an electrostatic force in the direction of the electric field, and a negative charge experiences an electrostatic force in the opposite direction of the electric field.
- The strength of the electric field E [N/C] created by a point charge with a charge Q [C] at a distance r [m] is given by Coulomb's law : **$E = k|Q|/r^2$**
- When there are several point charges, their respective electric fields can be added using the superposition principle. The direction of the electric field is radially **outwards** from the point charge when Q is positive, and radially **inwards** towards the point charge when Q is negative.



Equation box

Field and force	$F = qE \Rightarrow E = \frac{F}{q}$ (unit: N/C)
Point-charge field	$E = \frac{k\|Q\|}{r^2}$ (Coulomb's law; outward if $Q > 0$, inward if $Q < 0$)
Superposition	$E = E_1 + E_2 + \ldots$ (vector sum of the separate fields)

🛠 Worked Example

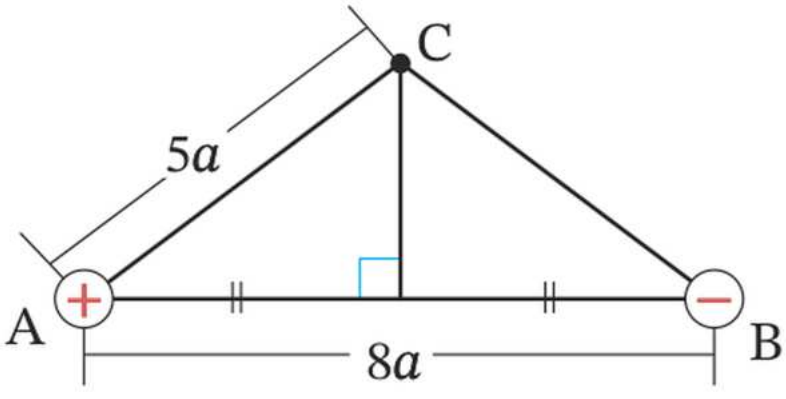


Figure 2-2-5 · Worked Example (b), from the original: the geometry alone — $+q$ at A and $-q$ at B a distance $8a$ apart, with C on the perpendicular bisector $5a$ from each.

Situation	Determine the following. (a) When a point charge of -4.0×10^{-10} C was placed at a certain location, it experienced an electrostatic force of 8.0×10^{-8} N to the right. Determine the direction and strength E [N/C] of the electric field at this location. (b) The distance between two points A and B is $8a$ [m]. A point charge of $+q$ [C] is placed at A and a point charge of $-q$ [C] is placed at B. Determine the direction and strength E_C [N/C] of the electric field at point C on the perpendicular bisector of AB, where AC = BC = $5a$ [m]. Take the proportionality constant of Coulomb's law as k [N·m²/C²].
Given and required	(a) $q = -4.0\times10^{-10}$ C, $F = 8.0\times10^{-8}$ N (to the right); find the direction and strength E. (b) $+q$ at A, $-q$ at B, AB = $8a$, AC = BC = $5a$, constant k; find the direction and strength E_C at C.
Representation	For (a), the charge is negative, so the force and the field point in opposite directions. For (b), see Figure 2-2-6: because AC = BC = $5a$ and half of AB is $4a$, the perpendicular distance from C to AB is $3a$ (a 3–4–5 right triangle), and E_A, E_B form two sides of a rhombus.
Strategy	Use $F = qE$ with magnitudes only, and fix the direction from the sign of the charge. For the two-charge case, find each point-charge field from $E = k\lvert Q\rvert/r^2$, then add the two fields as vectors (superposition).

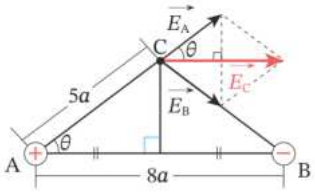


Figure 2-2-6 · Worked Example: at C the field of the positive charge at A and that of the negative charge at B are equal in size, so their vertical parts cancel and the resultant E_C lies parallel to AB.

Annotated solution	**(a)** The point charge is negative, so it feels a force opposite to the field. The force is to the right, so the **field points to the left**. Using $F = qE$ with magnitudes only, $8.0\times10^{-8} = (4.0\times10^{-10}) \times E$ $E = (8.0\times10^{-8})/(4.0\times10^{-10}) =$ **2.0×10^{2} N/C** (directed to the left). **(b)** The field E_A created at C by the charge at A points from A to C, with strength $k\cdot q/(5a)^2$. The field E_B created at C by the charge at B points from C to B, with strength $k\cdot q/(5a)^2$. These two equal fields form two sides of a rhombus, so their sum E_C points from A to B. Let $\angle CAB = \theta$. By symmetry $E_C = E_A \cos\theta \times 2$. Since $\cos\theta = 4a/5a = 4/5$, $E_C = k\cdot q/(5a)^2 \times (4/5) \times 2 =$ **$8kq/(125a^2)$ [N/C]**, directed from A to B.
Reasonableness check	(a) A field of 2.0×10^{2} N/C acting on 4.0×10^{-10} C gives $(2.0\times10^{2})(4.0\times10^{-10}) = 8.0\times10^{-8}$ N, which returns the given force. (b) Both charges have the same magnitude and the same distance $5a$ to C, so their fields are equal; the components perpendicular to AB cancel and the components along AB add, leaving a resultant along AB — consistent with $8kq/(125a^2)$.
Explain it yourself	In one sentence, explain why the field in part (a) points opposite to the force, and why in part (b) the resultant field at C lies along the line AB rather than perpendicular to it.

✍️ **Now you try:** A point charge of $+5.0\times10^{-9}$ C is placed where the electric field is 3.0×10^{3} N/C to the right. Find the magnitude and direction of the force on it.

🔆 Try

1. When a point charge of 2.0×10^{-9} C was placed at a certain location, it experienced an electrostatic force of 4.0×10^{-6} N to the right. Determine the direction and strength of the electric field at this location.
2. When a point charge of -3.0×10^{-6} C is placed at a position where the electric field is 1.6×10^{3} N/C to the right, determine the magnitude and direction of the electrostatic force the charge experiences.
3. Determine the strength of the electric field, in N/C, created by a point charge of 5.0×10^{-8} C at a point P that is 0.30 m away. Also, determine whether the direction of the electric field is directed radially outward from or inward toward the point charge. Take the proportionality constant of Coulomb's law as 9.0×10^{9} N·m^2/C^2.

4. As shown in the figure on the right, the distance between two points A and B is $2a$ [m]. A point charge of $-q$ [C] is placed at A and a point charge of $+q$ [C] is placed at B. Determine the direction and strength E_C [N/C] of the electric field at point C on the perpendicular bisector of AB such that the distance from the midpoint of AB is a [m]. Take the proportionality constant of Coulomb's law as k [N·m^2/C^2]; the answer may be left in radical form.

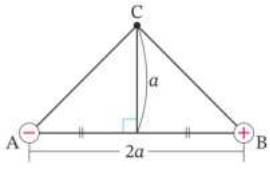


Figure 2-2-7 · Try 4: a point charge $-q$ at A and $+q$ at B, a distance $2a$ apart.

🌐 In a new context

Laser printers and photocopiers steer ink with electric fields. Fine toner grains are given a small electric charge and then released near a rotating drum that carries a pattern of charge. The field around the drum exerts a force $F = qE$ on each grain, pulling the charged toner exactly onto the regions that will become the printed image before the page is heated to fix it. The same law that describes a balloon lifting paper places millions of ink grains with precision.

Figure 2-2-8 · Inside a laser printer, fine toner powder grains are attracted selectively onto charged bands of the rotating drum and then transferred as a matching pattern onto the sheet of paper passing beneath.

➤ **Your turn:** A toner grain carries a charge of 2.0×10^{-12} C and sits where the field near the drum is 1.0×10^{5} N/C. Using $F = qE$, find the electrostatic force on the grain.

🧪 Nature of Science

From the original — lightning is also a discharge of static electricity. Lightning is a "discharge" in which static electricity built up in clouds suddenly flows through the air. Ice particles in the clouds rub against each other and generate static electricity, and negative charges gather at the base of the clouds. This strongly attracts positive charge on the ground; the electric field between cloud and ground grows so large that it breaks down the air, and a huge current flows — this is lightning.

Figure 2-2-9 · Negative charge collects at a cloud's base and positive charge gathers on the ground; the field between them grows strong enough to break down the air, and a lightning discharge follows the path.

The electric field is a scientific *model*: we cannot see the "property of space" itself, yet by defining it through the force on a unit positive charge we can predict, with a single equation, the force on any charge anywhere. A model earns trust when an unseen idea like a field makes measurable, testable predictions — such as where and when the air will break down in a storm.

Faraday drew his lines of force in the 1830s with almost no mathematics, and some dismissed the picture for exactly that reason; Maxwell then wrote it as equations, and it became the standard language of the subject. A physical picture can arrive before its mathematics and still be the right picture.

Your turn: Suggest one everyday observation that would let you tell, without touching a charged object, in which direction its electric field points.

Lesson question, answered — A charge does not reach across empty space directly; instead it fills the surrounding space with an electric field, and any second charge responds to the field where it sits. The field of a point charge is $E = k|Q|/r^2$ (outward for positive Q, inward for negative Q), and it exerts a force $F = qE$ — attractive or repulsive depending on the signs — which is why separated charges act on one another.

The scene we opened with.

📋 Exercise

1. When a point charge of -2.0×10^{-10} C was placed at a certain location, it experienced an electrostatic force of 6.0×10^{-8} N to the right. Determine the direction and strength of the electric field at this location.
2. When a point charge of 2.0×10^{-6} C is placed at a position where the electric field is 1.4×10^{3} N/C to the right, determine the magnitude and direction of the electrostatic force the charge experiences.

3. Determine the strength of the electric field, in N/C, at a point P that is 2.0 m away from a point charge of 8.0×10^{-6} C. Also, determine whether the direction of the electric field directed radially outward from or inward toward the point charge. Take the proportionality constant of Coulomb's law as 9.0×10^{9} N·m²/C².

4. The distance between two points A and B is 0.60 m. A point charge of 2.0×10^{-8} C is placed at A and a point charge of -2.0×10^{-8} C is placed at B. Let C be a point on the perpendicular bisector of AB whose distance from the midpoint O of AB is 0.30 m. Take the proportionality constant of Coulomb's law as 9.0×10^{9} N·m²/C² and $\sqrt{2} = 1.4$.

   (i) Determine the direction and strength E_O [N/C] of the electric field at point O.

   (ii) Determine the direction and strength E_C [N/C] of the electric field at point C.

   (iii) When the charge at B is changed to a point charge of 2.0×10^{-8} C, determine the direction and strength E_C [N/C] of the electric field at point C.

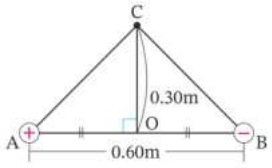


Figure 2-2-10 · Exercise 4: charges 2.0×10^{-8} C at A and -2.0×10^{-8} C at B, 0.60 m apart, with C on the perpendicular bisector.

5. In an equilateral triangle ABC of side 0.30 m, a point charge of 4.0×10^{-8} C is placed at A and a point charge of -4.0×10^{-8} C is placed at B. Determine the direction and strength E_C [N/C] of the electric field at point C. Take the proportionality constant of Coulomb's law as 9.0×10^{9} N·m²/C².

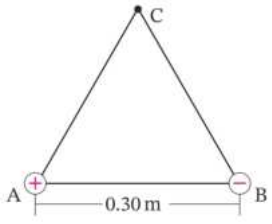


Figure 2-2-11 · Exercise 5: an equilateral triangle ABC of side 0.30 m with 4.0×10^{-8} C at A and -4.0×10^{-8} C at B.

6. The distance between two points A and B is $8a$ [m]. A point charge of $-q$ [C] is placed at A and a point charge of $+q$ [C] is placed at B. Determine the direction and strength E_C [N/C] of the electric field at point C on the perpendicular bisector of AB, where AC = BC = $5a$ [m]. Take the proportionality constant of Coulomb's law as k [N·m²/C²].

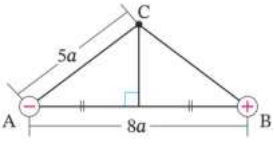


Figure 2-2-12 · Exercise 6: a charge $-q$ at A and $+q$ at B a distance $8a$ apart, with C off the line joining them.

2-3 Electric Potential

❓ **Lesson question:** Just as gravity gives an object potential energy by its height, what property of space gives a charge its potential energy?

You will learn to

1. State that the electric potential at a point is the potential energy of a unit positive charge placed there, measured in volts (V).
2. Calculate the potential energy of a charge from $U = qV$, in joules (J).
3. Find the work done by the electrostatic force when a charge moves between two points, $W = qV_A - qV_B$.

🔍 The phenomenon — predict before you continue

Shuffle across a carpet on a dry day, then reach for a metal door handle: a tiny spark leaps to your fingertip with a snap. Scuffing your feet did work to pile extra charge onto you, lifting it to a high electric potential; as your hand nears the grounded handle, that charge rushes across the gap to a place of much lower potential and gives up its stored energy as the spark. A charge, like a raised weight, carries energy set by where it sits in the field.

★ **Key idea**

Electric potential V — the electrostatic potential energy per unit positive charge, measured in volts (V); 1 V = 1 J/C.

Figure 2-3-1 · A spark jumps from a charged fingertip to a grounded door handle: charge lifted to a high potential releases its stored energy as it falls to a lower one.

Predict

(a) If you carry a small positive charge from a point of high potential to a point of low potential, does the electrostatic force do positive or negative work?

(b) If instead you push that positive charge toward a point of higher potential, who does the work — the field or your hand?

🔍 Explore — Charting potential across a conductive sheet

What you'll do: Measure how the voltmeter reading changes step by step across the sheet, and uncover that the electric field is strongest where the potential changes fastest with distance.

Time: 20 min

You will need:

- Conductive paper with two electrodes
- Battery (a few volts)
- Voltmeter with two probes
- Ruler

Conductive paper

Battery

Voltmeter

Ruler

Steps

1. Connect the two electrodes on the conductive paper to the battery, so a steady electric field spreads across the sheet.
2. Touch one voltmeter probe to electrode A as a fixed reference; lay the ruler along the straight line joining the two electrodes.
3. Move the second probe in equal steps of distance x along that line and record the voltmeter reading V at each step; for each interval compute the change in potential per unit distance, $\Delta V/\Delta x$.
4. Now hunt sideways for other points that give the same reading and join them — an equipotential line.

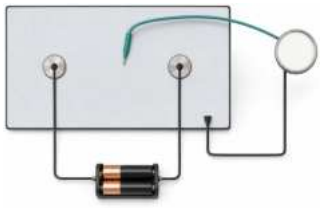


Figure 2-3-2 · Two electrodes on conductive paper are wired to a battery; a voltmeter keeps one probe on a fixed reference while the other is moved over the sheet to find points of equal potential.

Run	Distance x (cm)	Voltmeter reading V (V)	$\Delta V/\Delta x$ (V/cm)
1			
2			
3			

> ★ **Working note**
> Press the reference probe firmly on electrode A and leave it there for every reading; a loose contact shifts the whole column. Read V with the moving probe held still and vertical.

Analyze what you collected:

(a) Read down the $\Delta V/\Delta x$ column: does the potential rise by the same amount every step, or faster in one region than another? Where is $\Delta V/\Delta x$ largest?

(b) The electric field is strongest where $\Delta V/\Delta x$ is largest. From your table, predict where equipotential lines would crowd most closely, and explain why sliding a charge *along* an equipotential needs no work.

✎ Write your explanation in two lines before you read the next page.

From observation to model

Why the drawing rules are not conventions

A picture of a field is only useful if it cannot lie, and each rule for drawing one turns out to be forced rather than chosen. At any given point the arrangement of charges pushes a probe one way and one way only; two curves meeting there would offer that probe two directions to follow at the same instant, which no measurement has ever shown, and a curve that split in two would do the same. Spacing is settled just as firmly. Follow every curve leaving a small sphere around a charge and the same

set of them must spread over the surface of a larger sphere farther out — the same supply thinned across a wider area is exactly what a weakening push means. So crowding is not a decorative hint about strength; it is strength, drawn.

Dividing out the probe a second time

The previous lesson removed a probe charge from a force to leave something belonging to the point itself. Energy invites exactly the same move. Carry a probe to a chosen point and energy is stored in it; carry twice as much charge along the same path and twice as much energy is stored, so the ratio of stored energy to charge is again independent of what was carried, and can be tabulated once for the point alone. Because energy is a plain number with no direction, this ratio is a scalar — a considerable convenience, since two of them combine by ordinary addition, with none of the vector construction that adding two fields demands.

Why only differences can be measured

The rest follows in two lines. The electrostatic force is conservative, so the work it does while a charge travels equals the stored energy lost on the way — start minus finish, by whatever route. Write the stored energy at each end as charge times the ratio just defined, and the charge is common to both terms and factors straight out, leaving that charge multiplied by the drop between the two ends (Figure 2-3-3). One consequence deserves attention: the drop is all that appears, never the two values separately. Whatever they share cancels, so the zero may be placed wherever it is convenient — at the ground, at a battery terminal, infinitely far away — without altering a single measurable prediction. It is also why sliding a charge along a curve of constant value costs nothing at all.

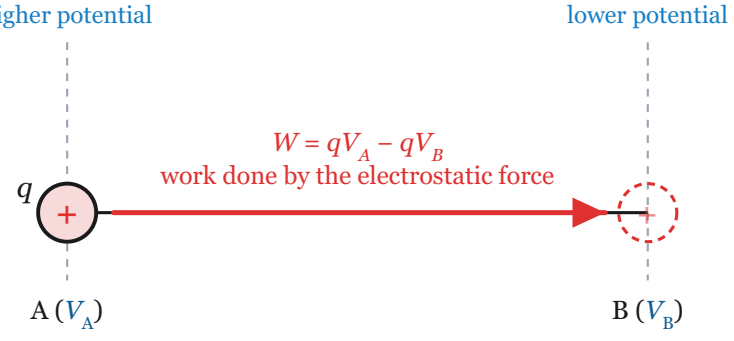


Figure 2-3-3 · The charge is released at the higher end of the ladder and arrives at the lower one. What it gives up in stored energy is exactly what the field has spent pushing it along, and only the height of the step between the two ends enters the sum.

★ **Note**

These curves are a diagram, not a discovery. Nothing is threaded through the space and nothing travels along them. Draw twice as many and the arrangement is unchanged — only the *relative* crowding from place to place carries meaning.

★ **Key fact**

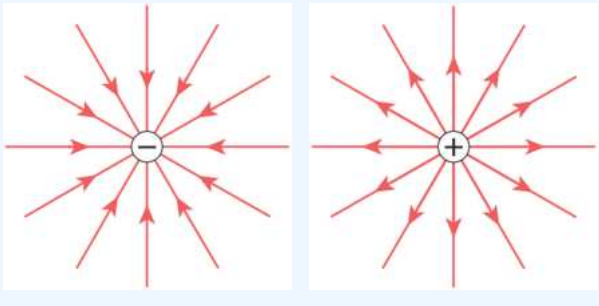
1 J/C is what one volt means, so a 1.5 V cell hands over 1.5 J to every coulomb it drives around a circuit. One volt is a modest amount of energy per charge: a lightning stroke bridges hundreds of millions of them.

★ **Term**

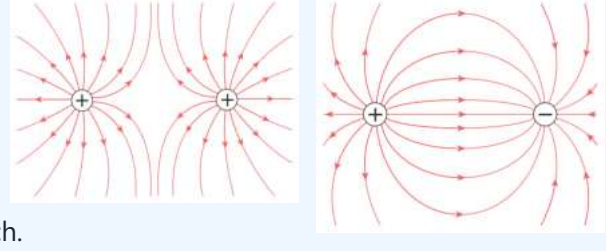
Where the picture stops — both the curves and the equal-value contours describe a static arrangement. Let the charges themselves move and the whole pattern must be redrawn, so the tidy still image of this lesson is an equilibrium snapshot rather than a permanent map of the space.

Point

! A line representing the state of an electric field is called an **electric field line**. Electric field lines are drawn with arrows indicating the direction.



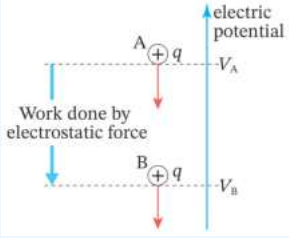
! Properties of electric field lines
- Start from **positive charges** and end at **negative charges**.
- Do not intersect or branch.



• Are closer together where the electric field is stronger.
The tangent to the field line at any point gives the direction of the electric field.

! The potential energy possessed by a unit positive charge when it is placed at a certain point is called **electric potential**, and its unit is the **volt** (symbol **V**). The potential energy U [J] due to the electrostatic force possessed by a charge q [C] can be expressed as $U = qV$, where V [V] is the electric potential.

! When a charge of q [C] moves from point A (potential V_A) to point B (potential V_B), the work done by the electrostatic force is equal to the amount of decrease in potential energy due to the electrostatic force.
$W =$ $qV_A - qV_B$



Equation box

Electric potential energy	$U = qV$ (U in J, q in C, V in V)
Work by electrostatic force	$W = qV_A - qV_B = q(V_A - V_B)$

Worked Example

Situation	Determine the following. (a) A point charge of -2.0×10^{-5} C is placed at a position where the electric potential is 5.0 V. Determine the potential energy U in J due to the electrostatic force possessed by this point charge. (b) A point charge of 4.2×10^{-8} C placed at point A moved to point B, where the electric potential is 15 V lower than at point A. Determine the work W in J done by the electrostatic force. (c) An external force is applied to a point charge of -3.0×10^{-7} C placed at point A, and it is slowly carried to point B, where the electric potential is 2.0 V lower than at point A. Determine the work W done by the electrostatic force and the work W' done by the external force in J, respectively.
Given and required	(a) $q = -2.0\times10^{-5}$ C, $V = 5.0$ V; find U. (b) $q = 4.2\times10^{-8}$ C, $V_A - V_B = 15$ V; find W. (c) $q = -3.0\times10^{-7}$ C, $V_A - V_B = 2.0$ V; find W and the external-force work W'.
Representation	See Figure 2-3-3 for a charge moving from A to B. Figure 2-3-5 shows state (c): the potential at B is 2.0 V below the potential at A, and the external force balances the electrostatic force throughout the slow carry.

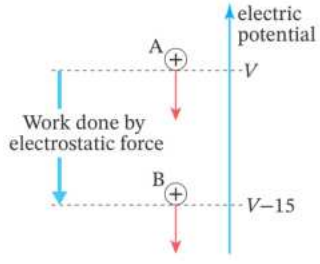


Figure 2-3-4 · Worked Example (b): a positive charge falls from A at potential V to B at $V - 15$, and the electrostatic force does the work.

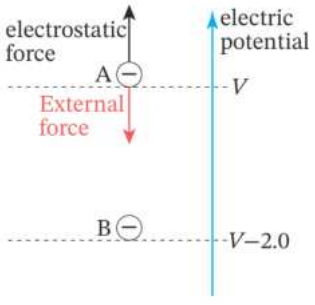


Figure 2-3-5 · Worked Example (c): a negative charge is carried slowly from A, at potential V, to B, at potential $V - 2.0$ V; the external force balances the electrostatic force.

Strategy	Two questions settle every part. Is the charge sitting still, or is it going somewhere? A charge at rest calls for its stored energy, $U = qV$; a charge that travels calls for the drop it falls through, $q(V_A - V_B)$. Read off the two end values first, subtract in the order start-minus-finish, and keep the sign of the charge — a negative charge reverses the answer.
Annotated solution	**(a)** $U = qV = (-2.0\times10^{-5})\times5.0 =$ **-1.0×10^{-4} J**. **(b)** Let the potential at A be V [V], so the potential at B is (V − 15) [V]. The potential energy at A is $(4.2\times10^{-8})\times V$ and at B is $(4.2\times10^{-8})\times(V - 15)$, so $W = (4.2\times10^{-8})\times V - (4.2\times10^{-8})\times(V - 15) = (4.2\times10^{-8})\times15 =$ **6.3×10^{-7} J**. **(c)** Let the potential at A be V [V], so B is at (V − 2.0) [V]. For the electrostatic force, $W = (-3.0\times10^{-7})\times V - (-3.0\times10^{-7})\times(V - 2.0) = (-3.0\times10^{-7})\times2.0 =$ **-6.0×10^{-7} J**. Since the charge is "slowly carried," the external force balances the electrostatic force, so the external-force work is $-W$. Therefore $W' =$ **6.0×10^{-7} J**.
Reasonableness check	(a) A negative charge at a positive potential has negative potential energy, as found. (b) A positive charge falling to a lower potential loses potential energy, so the electrostatic force does positive work — the sign is correct. (c) The electrostatic force opposes the slow carry, so it does negative work, and the balancing external force does an equal and opposite amount, $W' = -W$.
Explain it yourself	In one sentence, say why the unknown value V at A never had to be found in parts (b) and (c) — and what that tells you about where a physicist is free to put the zero.

✍️ **Now you try:** A point charge of 5.0×10^{-6} C at point A moves to point B, where the electric potential is 8.0 V lower than at A. Find the work W done by the electrostatic force.

Try

1. A point charge with a charge of 1.2×10^{-5} C is placed at a position where the electric potential is 6.0 V. Determine the potential energy U in J due to the electrostatic force possessed by this point charge.
2. A point charge with a charge of -3.6×10^{-8} C placed at point A moved to point B, where the electric potential is 20 V lower than at point A. Determine the work W in J done by the electrostatic force.
3. An external force is applied to a point charge of 2.1×10^{-7} C placed at point A, and it is slowly carried to point B, where the electric potential is 10 V higher than at point A. Determine the work W done by the electrostatic force and the work W' done by the external force in J, respectively.

🌐 In a new context

A car battery is marked 12 V. That number is a potential difference: every coulomb of charge that the battery drives from its positive terminal, through the headlights, to its negative terminal falls through 12 V of potential and hands over 12 J of energy to the lamp. The very same relation $W = q(V_A - V_B)$ that governs one microscopic point charge also fixes how much energy a whole current delivers around a circuit.

➤ **Your turn:** A charge of 2.0 C is carried by a 12 V battery from the high-potential terminal to the low-potential terminal. Using $W = q(V_A - V_B)$, find the work done by the electrostatic force.

Figure 2-3-6 · A 12-volt car battery drives current around a complete circuit to light the headlamp, which glows and casts a forward beam while the wires and battery themselves stay dark.

🧪 Nature of Science

From the original — contour lines on a map and equipotential lines. Lines connecting points of equal electric potential are called equipotential lines. These are exactly like the contour lines on a map. Just as dense contour lines mean a steep slope, dense equipotential lines mean a strong electric field. And just as water flows from a higher place to a lower place, a positive charge moves along the electric field from a higher potential to a lower potential.

This is a scientific analogy: a familiar picture — height on a landscape — is borrowed to make an invisible quantity, electric potential, easy to reason about. An analogy is a modeling tool, powerful because it lets known intuition carry over, but it must be checked at its limits, since potential and height are not the same physical quantity.

Galvani read the twitch of a frog's leg in 1791 as electricity belonging to the animal; Volta insisted it came from the contact of two metals, and closed the argument in 1800 by building a pile of them that worked with no animal at all. A dispute in physics is settled less by argument than by an apparatus that answers it.

Your turn: Suggest one everyday map or diagram, other than a contour map, that uses lines or shading to show where a quantity is large or small, and say what its "equipotential lines" would represent.

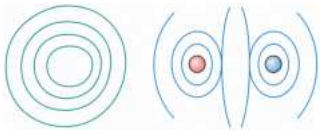


Figure 2-3-7 · Contour lines on a hill and equipotential lines around two charges obey the same rule: where the lines crowd together, the slope is steep and the field is strong.

Lesson question, answered — The property of space that fixes a charge's potential energy is the electric potential V: it is the potential energy of a unit positive charge at that point, so a charge q there has energy $U = qV$. Moving the charge from A to B, the electrostatic force does work equal to the drop in this energy, $W = qV_A - qV_B$ — exactly the role that height plays for gravity.

The scene we opened with.

📋 Exercise

1. A point charge with a charge of 6.0×10^{-6} C is placed at a position where the electric potential is 2.0 V. Determine the potential energy U in J due to the electrostatic force possessed by this point charge.
2. A point charge with a charge of -8.0×10^{-5} C is placed at a position where the electric potential is 1.5 V. Determine the potential energy U in J due to the electrostatic force possessed by this point charge.
3. A point charge with a charge of 2.1×10^{-8} C placed at point A moved to point B, where the electric potential is 3.0 V lower than at point A. Determine the work W in J done by the electrostatic force.
4. A point charge with a charge of 3.2×10^{-7} C placed at point A moved to point B, where the electric potential is 2.0 V higher than at point A. Determine the work W in J done by the electrostatic force.

5. An external force is applied to a point charge of -3.2×10^{-7} C placed at point A, and it is slowly carried to point B, where the electric potential is 5.0 V lower than at point A. Determine the work W done by the electrostatic force and the work W' done by the external force in J, respectively.

2-4 Uniform Electric Field and Potential Difference

❓ **Lesson question:** In a uniform electric field, how are the potential difference between two points, the field strength, and the distance between them related?

You will learn to

1. Relate the potential difference to the field strength and the distance in a uniform electric field using $V = Ed$.
2. Read the electric field strength E as the magnitude of the slope of a position–potential (x–V) graph, in V/m.
3. Determine the direction and magnitude of the electrostatic force $F = qE$ on a charge placed in the field.

🔍 The phenomenon — predict before you continue

Two large parallel metal plates are connected to a battery, one plate positive and one negative. Release a tiny positively charged bead anywhere in the gap and it drifts toward the negative plate — but the striking thing is what happens when you release it somewhere else. Near the top plate, near the bottom, off to one side: the push is the same size and points the same way every time. Map the potential step by step from one plate to the other and it slides down at a steady rate, high beside the positive plate and low beside the negative one.

★ **Key idea**

Field strength E [V/m] between two parallel plates d apart, at a potential difference V. Is it the same everywhere?

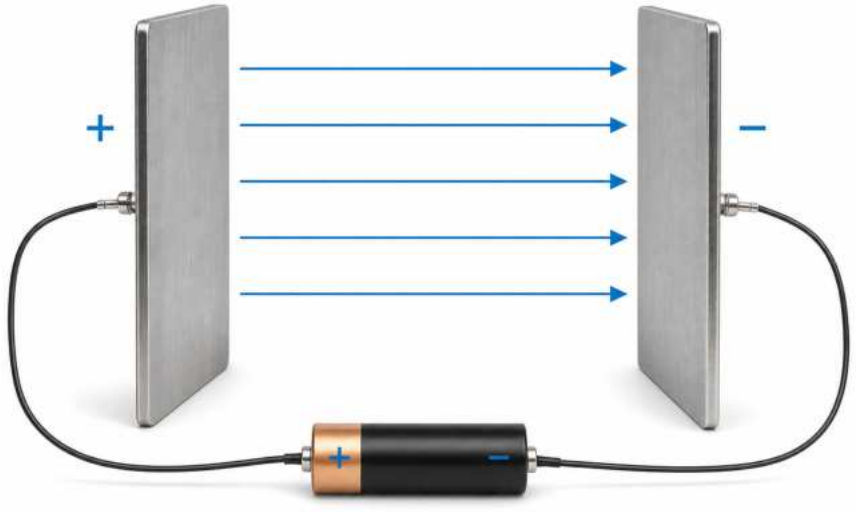


Figure 2-4-1 · Between two parallel charged plates the field is uniform — the same strength and direction at every point.

Predict

(a) As you move from the positive plate to the negative plate, does the electric potential rise, fall, or stay constant?

(b) If you double the plate separation while keeping the same voltage across the plates, does the field strength rise, fall, or stay the same?

🔍 Explore — Mapping potential between two parallel electrodes

What you'll do: Read the potential at several positions between two straight parallel electrodes on conductive paper — a flat stand-in for a parallel-plate gap — and test whether the field $E = V/x$ is the same everywhere.

Time: 15 min

You will need:

- Conductive paper with two straight parallel electrodes
- Steady (DC) voltage supply
- High-resistance voltmeter with two probes
- Ruler to read the probe position

Conductive paper

DC supply

Voltmeter

Ruler

Steps

1. Fix the electrode separation d and set a steady voltage across the two electrodes; leave it unchanged for the whole run.
2. Hold one voltmeter probe on the negative electrode as a fixed reference; touch the other probe to the paper at a position x measured from that electrode and record the potential V.
3. Repeat at other positions, and for each row work out the derived column $E = V/x$ in V/m.
4. Plot V against position x and look at the shape of the graph.

Run	Position x (m)	Potential V (V)	$E = V/x$ (V/m)
1			
2			
3			

Analyze what you collected:

(a) Look down the last column: is $E = V/x$ nearly the same for every position, and do equal steps in x give equal steps in V?

(b) The graph of V against x is a straight line — what does the magnitude of its slope tell you about the field strength, and which way does the field point?

✎ Write your explanation in two lines before you read the next page.

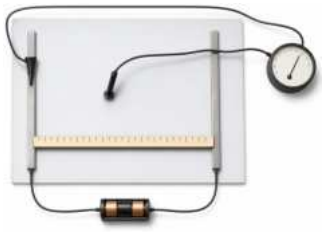


Figure 2-4-2 · Two straight parallel electrodes on conductive paper, a fixed distance d apart, are held at a steady voltage by a DC supply; the voltmeter keeps one probe on the negative electrode while the other reads the potential at each position x between them.

★ Working note

Measure x from the negative (low-potential) electrode along the straight line to the positive one, and keep the moving probe on that center line so each reading belongs to one clean position.

From observation to model

Where uniformity comes from, and what it buys

Nothing about a field forces it to be the same everywhere; around a single charge it plainly is not, weakening as the square of the distance. Two large parallel plates are the arrangement that removes the variation. Stand anywhere in the middle of the gap and the plates look alike in every direction — no nearer edge, no preferred spot — so there is nothing left for the field to vary *with*. That sameness is the whole payoff of this lesson: it converts the relations of the last two lessons from statements about points into statements about lengths you can measure with a ruler.

Deriving the relation in three steps

Take the plate arrangement and push a charge q straight across the gap, a distance d, in the direction the field points. Now count the energy twice over. The last lesson gives one count: the electrostatic force gives up q times the drop between the two ends. Mechanics gives the other: a steady force $F = qE$ acting along a straight run of length d does work qEd. The two counts describe the same transfer, so $q(\text{drop}) = qEd$, and the charge cancels — as it must, since neither the plates nor the gap know what was sent across. What is left connects three quantities that a voltmeter and a ruler can both reach.

Why the graph is a straight line

Equal steps of distance now cost equal amounts of potential, because the same field strength is doing the same work over each step. A quantity that changes by equal amounts over equal intervals plots as a straight line, so a straight x–V graph is the visual signature of uniformity — and where a real graph bends, the field is not uniform there. Its steepness measures how much potential each meter costs, which is precisely the field strength; and since traveling with the field means giving up potential, the line runs downhill in the direction the field points. That is why the sign of the gradient tells you the direction and its size tells you the strength (Figure 2-4-3).

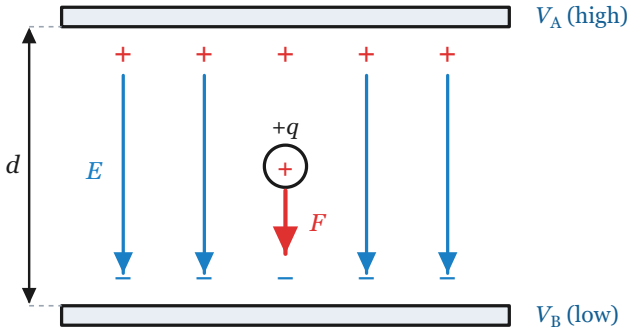


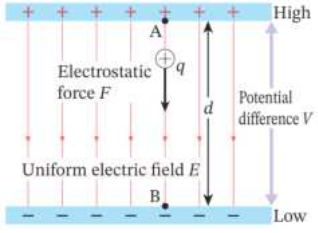
Figure 2-4-3 · A uniform field E between two horizontal plates a distance d apart runs from the high-potential plate V_A to the low-potential plate V_B; a positive charge $+q$ in the gap feels a force $F = qE$ from high toward low potential.

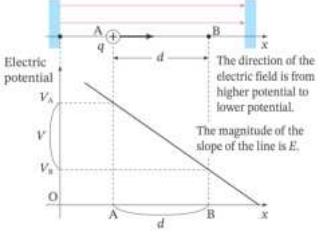
> ★ **Term**
> The sameness argument fails near the rim, where a point can tell which edge it is closest to. There the lines bow outward and spread, so the plates must be wide compared with their separation before any calculation in this lesson may be trusted.

> ★ **Key fact**
> Because the relation can be read either way round, the field's unit can be written either way round too: as force per charge (N/C) or as potential drop per length (V/m). One newton per coulomb and one volt per meter are the same size.

Point

- The difference in electric potential between two points, $V_A - V_B$, is called **potential difference (voltage)**.
- An electric field that has the same strength and direction everywhere in space is called a uniform electric field. In a uniform electric field of strength E [N/C], the potential difference V [V] between A and B separated by a distance d [m] along the direction of the electric field is **$V = Ed$**.
- From $V = Ed$, it can be expressed as $E = V/d$, so in a graph showing the relationship between position x [m] and electric potential V [V], the **electric field strength E** becomes the slope of the straight line. Also, **V/m** is used as the unit of electric field strength. The electric field points from **higher** to **lower** electric potential.




Electric potential
The direction of the electric field is from higher potential to lower potential.
The magnitude of the slope of the line is E.

Equation box

Potential difference	$V = V_A - V_B$ (unit: V)
Uniform field	$V = Ed \Longrightarrow E = \frac{V}{d}$ (unit: V/m = N/C)

Force on a charge	$F = qE$ (unit: N); force on $+q$ is along E

🛠 Worked Example

Situation	There is a uniform electric field parallel to the x-axis. The graph of the electric potential V [V] against the position coordinate x [m] is shown in Figure 2-4-4 (the potential is 20 V at $x = 0$ and falls to 0 at $x = 0.50$ m). (a) Determine the direction of the electric field. (b) Determine the electric field strength E in V/m. (c) When a charge of $+1.2\times10^{-7}$ C is placed in this electric field, determine the direction and magnitude F [N] of the force experienced by the charge.
Given and required	From the graph: $V = 20$ V at $x = 0$ and $V = 0$ at $x = 0.50$ m, so the potential difference over $d = 0.50$ m is 20 V. Charge $q = +1.2\times10^{-7}$ C. Find (a) the field direction, (b) E in V/m, (c) the direction and magnitude of F.
Representation	Read the straight line on the x–V graph (Figure 2-4-4). The potential is higher at $x = 0$ and lower at $x = 0.50$ m.
Strategy	Steps to solve uniform-field problems: from the field strength E and the distance d, relate them to the potential difference V by $V = Ed$, so $E = V/d$ is the magnitude of the slope. Use the property that the field points from higher toward lower potential. The force on a charge is $F = qE$, and for a positive charge it points along the field.
Annotated solution	**(a)** The potential falls as x increases (20 V at $x = 0$ down to 0 at $x = 0.50$ m), so the field points from higher to lower potential — the **positive direction of the x-axis**. **(b)** $E = V/d = 20 / 0.50 =$ **40 V/m**. **(c)** A positive charge feels a force in the same direction as the field, so the force is in the **positive x-direction**. Its magnitude is $F = qE = (1.2\times10^{-7}) \times 40 =$ **4.8×10^{-6} N**.
Reasonableness check	The slope 20 V over 0.50 m gives 40 V/m, a modest field. Multiplying by the tiny charge 1.2×10^{-7} C gives a force of order 10^{-6} N, which is sensible for so small a charge. The positive charge is pushed toward the low-potential end, as expected.

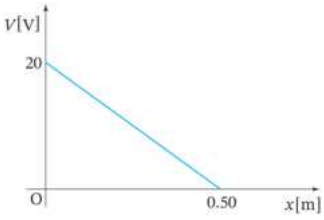


Figure 2-4-4 · Worked Example: the potential V falls uniformly from 20 V at $x = 0$ to 0 at $x = 0.50$ m.

🧐 Explain it yourself	In one sentence, say why the field strength equals the magnitude of the slope of the x–V line, and why the force on a positive charge points toward lower potential.

✍️ **Now you try:** A uniform field parallel to the x-axis has potential V = 30 V at x = 0 falling to 0 at x = 0.60 m. Find the field strength and the force on a $+2.0\times10^{-7}$ C charge.

🔆 Try

1. Points A and B are on the same electric field line in a uniform electric field. The distance between A and B is 0.80 m, and the electric potential at A is 40 V higher than at B. Determine the strength of this electric field in V/m.
2. There is a uniform electric field parallel to the x-axis. The graph representing the electric potential V [V] at position coordinate x [m] is shown in Figure 2-4-5.
   (i) Determine the direction of the electric field.
   (ii) Determine the electric field strength E in V/m.
   (iii) When a charge of $+1.6\times10^{-7}$ C is placed in this electric field, determine the direction and magnitude F [N] of the force experienced by the charge.

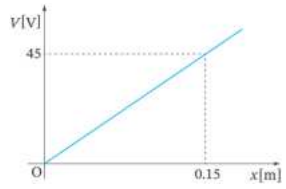


Figure 2-4-5 · Try: the potential V rises uniformly with x, reaching 45 V at x = 0.15 m.

🌐 In a new context

The space between the flat base of a thundercloud and the level ground below can sometimes be treated as a uniform electric field, with the same field strength throughout. From the height of the cloud base and the potential difference between the cloud and the ground, the field strength between them can be estimated with $E = V/d$. When that field grows past the limit that air can withstand — about 3×10^{6} V/m — the air suddenly conducts and a lightning strike occurs.

Figure 2-4-6 · The flat base of a storm cloud and the level ground below act like two parallel plates, so the nearly uniform electric field in the air gap sends a lightning bolt straight down when the air breaks down.

➤ **Your turn:** A thundercloud base is 800 m above flat ground, with a potential difference of 2.4×10^{8} V between cloud and ground. Estimate the field strength, and say whether it has yet reached the 3×10^{6} V/m breakdown limit of air.

🧪 Nature of Science

From the original — the field between a thundercloud and the ground. Treating the huge, ragged gap between a cloud and the ground as a clean "uniform electric field" is an idealization: the real field is far from perfectly even. Yet this simple model lets us estimate the field from just two measurable quantities, the potential difference and the separation, and it predicts the right order of magnitude for when breakdown occurs.

A good model in physics is judged by how usefully and how honestly it predicts, not by being exactly true. The uniform-field model is trusted where the plates or surfaces are large and close, and it is known to fail near edges, where the field spreads out and the lines are no longer straight and evenly spaced.

Millikan measured the elementary charge in 1909 by balancing oil drops in exactly this idealization — a uniform field between two flat plates — and the value he obtained stood. An idealization earns its keep by the measurement it makes possible.

Your turn: Suggest one situation where treating a real electric field as uniform would give a poor estimate, and say which feature of the real field breaks the assumption.

Lesson question, answered — In a uniform electric field the three quantities are tied together by $V = Ed$: the potential difference between two points equals the field strength times the distance between them measured along the field. Equivalently, $E = V/d$ is the magnitude of the slope of the x–V graph, in V/m, and the field points from higher toward lower potential. A charge in the field feels a force $F = qE$, along the field for a positive charge.

The scene we opened with.

Exercise

1. Points A and B are on the same electric field line in a uniform electric field. The distance between A and B is 0.60 m, and the electric potential at A is 15 V higher than at B. Determine the strength of this electric field in V/m.
2. Points A and B are on the same electric field line in a uniform electric field with a strength of 40 V/m. The electric potential at A is 20 V higher than at B. Determine the distance between A and B in m.
3. There is a uniform electric field parallel to the x-axis. The graph representing the electric potential V [V] at position coordinate x [m] is shown in Figure 2-4-7.
   (i) Determine the direction of the electric field.
   (ii) Determine the electric field strength E in V/m.
   (iii) When a charge of $+2.8\times10^{-7}$ C is placed in this electric field, determine the direction and magnitude F [N] of the force experienced by the charge.
4. There is an electric field parallel to the x-axis, and the graph representing the electric potential V [V] at position coordinate x [m] is shown in Figure 2-4-8.
   (i) Determine the electric field strength at point A in V/m.
   (ii) Determine the electric field strength at point B in V/m.
   (iii) When a charge of $+1.0\times10^{-7}$ C is carried from point B to the origin O, determine the work W in J done by the electric force.

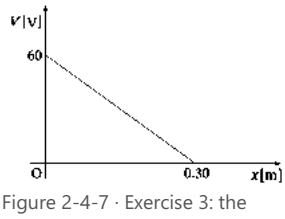


Figure 2-4-7 · Exercise 3: the potential V falls uniformly from 60 V at x = 0 to 0 at x = 0.30 m.

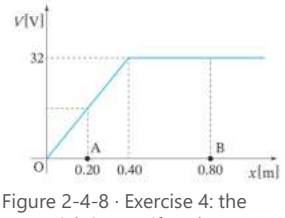


Figure 2-4-8 · Exercise 4: the potential rises uniformly to 32 V at x = 0.40 m and stays constant beyond it; A lies at x = 0.20 m and B at x = 0.80 m.

2-5 Motion of Charges Subjected to Electrostatic Force

❓ **Lesson question:** Just as a dropped ball speeds up under gravity, how do a charge's speed and energy change as it moves through an electric field?

You will learn to

1. Express the potential energy of a charge in an electric field as $U = qV$, in joules (J).
2. Apply conservation of mechanical energy, $\frac{1}{2}mv^2 + qV = \text{constant}$, to a charge acted on by the electrostatic force alone.
3. Find the speed of a charged particle at one point from its speed and the electric potentials at two points, keeping the signs of q and V.

🔍 The phenomenon — predict before you continue

A positive ion sits in a region of high electric potential and drifts toward a region of lower potential, pushed only by the electrostatic force. No hand touches it, yet it speeds up — much as a ball dropped from a height gains speed under gravity. The electric field does the work here, trading stored electrostatic potential energy for motion.

★ **Key idea**
The electrostatic force is conservative: the work it does between two points does not depend on the path taken.

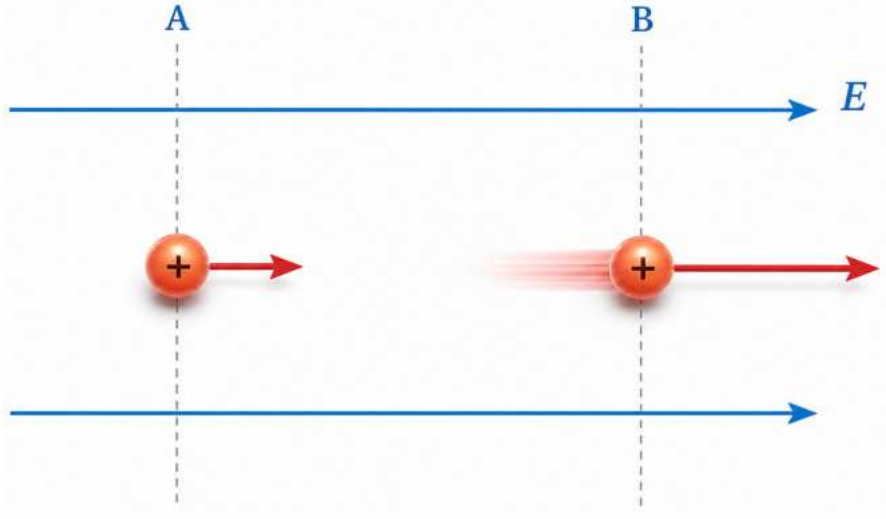


Figure 2-5-1 · A positive ion speeds up as it drifts from high potential A to lower potential B along the field E.

Predict

(a) As the positive ion moves from high to lower potential, does its kinetic energy rise, fall, or stay the same?

(b) As its potential energy qV falls, where does that energy go?

🔍 Explore — Energy trade on a smooth ramp

What you'll do: Roll a marble down a smooth ramp and measure the speed it gains for a measured drop in height, to test whether the speed it picks up is fixed by the drop alone.

Time: 15 min

You will need:

- A smooth, low-friction inclined ramp or track
- A marble (or small steel ball)
- A photogate with a timer to read speed at point B
- A ruler or meter stick, and a balance to find the marble's mass m

Inclined ramp

Photogate

Ruler

Balance

Steps

1. Weigh the marble to get its mass m, and set the photogate at a lower point B on the smooth ramp.
2. Release the marble from rest at a marked point A, measure the vertical drop Δh to B with the ruler, and read its speed v_B from the photogate.
3. For each run compute the kinetic energy gained, $\frac{1}{2}mv_B^2$, and the gravitational potential energy lost, $mg\Delta h$ (use $g = 9.8$ m/s^2).
4. Repeat from two higher release points so the drop Δh is larger each time.

Run	Drop Δh (m)	Speed v_B (m/s)	v_B^2 (m^2/s^2)	$v_B^2/\Delta h$ (m/s^2)
1				
2				
3				

Analyze what you collected:

(a) Look down the last column. Within measurement error, does $v_B^2/\Delta h$ stay the same as you release the marble from higher and higher? What does a constant $v^2/\Delta h$ tell you about where the energy the marble loses in falling has gone?

(b) A charge feels a conservative electrostatic force just as the marble feels gravity. If you replace the height term mgh with the electrostatic potential energy qV, what constant sum should a charge keep as it moves between two points in a field — and would it speed up or slow down as qV falls?

✎ Write your explanation in two lines before you read the next page.

Figure 2-5-2 · A marble released at height h_A (point A) slides down the smooth, low-friction ramp to the lower point B at height h_B; a photogate reads its speed at each point.

★ **Working note**

Release the marble from *rest* so its starting speed is zero, and take the drop height $\Delta h = h_A - h_B$ as the *vertical* fall between the two points, not the length of track along the slope. A ball that rolls carries part of its energy as spin, so the $\frac{1}{2}mv^2$ you measure comes out below $mg\Delta h$ — the same fraction of it every run. What is exact is the proportionality: double the drop and v^2 doubles with it.

From observation to model

Potential energy of a charge in a field

Why should energy be *charge* multiplied by *potential*? Because that is what potential already means: each coulomb placed at a point carries a definite store of energy, and the potential V is nothing other than the size of that store per coulomb. Multiply it by however many coulombs are actually sitting there and the store itself comes back, in joules. Nothing extra has been measured — the potential map of the field, drawn once, becomes an energy map the moment you name the charge you are placing on

it. And because a coulomb can be counted as negative and a potential can run below zero, the product carries two independent signs; a negative charge parked at a positive potential holds a *negative* store.

Why a ledger can be written at all

Gravity is granted a potential energy *mgh* for one reason: the work it does between two heights comes out the same whichever route the body takes. The electrostatic force earns its *qV* on identical grounds. That single property is what makes the whole calculation possible — the field's work between two points is fixed by the two potentials alone, so no share of the energy can be quietly spent on the length or shape of the path. Whatever the store gives up must therefore reappear as motion, and nothing else. Evaluate the two-term ledger once at the start and once at the finish, set the two evaluations equal, cancel the mass and the charge wherever they repeat, and the ledger rearranges into a formula for the unknown speed:

$$v_B^2 = v_A^2 + \frac{2q(V_A - V_B)}{m}$$

Read what is missing from it — no distance, no field strength, no description of the route. Only the two potentials and the speed you started with.

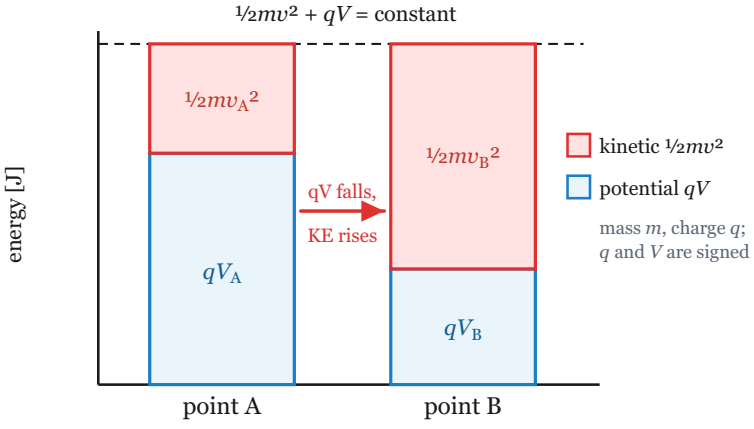
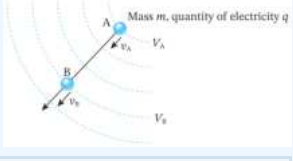


Figure 2-5-3 · Energy bars at points A and B: the electrostatic potential energy *qV* (blue) and the kinetic energy ½*mv*² (red) trade places, but their sum reaches the same constant total at both points, so as *qV* falls from A to B the kinetic energy rises by the same amount.

Point

- Law of conservation of mechanical energy
  (Potential energy) + (Kinetic energy) = constant
- The potential energy *U* [J] due to the electrostatic force possessed by a charge *q* [C] can be expressed as ***U = qV***, where *V* [V] is the electric potential.
- Since the electrostatic force is a conservative force, the law of conservation of mechanical energy holds.



	Kinetic energy	The potential energy due to the electrostatic force
A	$\frac{1}{2}mv_A^2$	qV_A
B	$\frac{1}{2}mv_B^2$	qV_B

$$\frac{1}{2}mv_A^2 + qV_A = \frac{1}{2}mv_B^2 + qV_B$$

· *q* and *V* must be signed

★ **Key fact**
The joule, read off the potential map: 1 J = 1 C·V. So a charge of one microcoulomb resting at one kilovolt already holds a millijoule — and an ion carrying only 10^{-19} C needs thousands of volts before its store is worth counting.

★ **Term**
Conservative force — one whose work between two points is identical along every path, which is precisely the condition for a potential energy to exist for it. Friction and air resistance fail that condition: their work grows with the length of the route, so no store can be defined for them, and the constant total breaks down the instant either one touches the charge. The model of this lesson therefore assumes a charge in vacuum, acted on by the field alone.

★ **Note**
Do not decide in advance which way the particle should speed up. A *negative* charge reverses the story and gains speed climbing *toward* higher potential, because for $q < 0$ the product *qV* shrinks as *V* grows. Substitute the signed values and let the algebra deliver the verdict.

📊 Equation box	
Electrostatic potential energy	$U = qV$ (unit: J; q in C, V in V)
Energy ledger	the two-term sum written out in the Point above takes one value for the whole journey — evaluate it at the start point, evaluate it at the end point, set the two equal
Speed at B	$v_B^2 = v_A^2 + \frac{2q(V_A - V_B)}{m}$ (the ledger solved for the unknown speed)

🛠 Worked Example

Situation	A positive ion with a mass of 6.6×10^{-27} kg and a charge of 3.2×10^{-19} C passed point A (electric potential 1.5×10^{3} V) at a speed of 2.0×10^{5} m/s. Determine the speed in m/s when this positive ion passes point B (electric potential -1.8×10^{3} V). Assume the positive ion is subjected only to the electrostatic force.
Given and required	$m = 6.6\times10^{-27}$ kg, $q = 3.2\times10^{-19}$ C, $V_A = 1.5\times10^{3}$ V, $v_A = 2.0\times10^{5}$ m/s, $V_B = -1.8\times10^{3}$ V; find v_B in m/s.
Representation	See Figure 2-5-3: the ion carries kinetic energy $\frac{1}{2}mv^2$ and electrostatic potential energy qV at each point, and the two sum to the same value at A and at B.
Strategy	Name the two points the ion visits and list what is known at each: a speed and a potential. Because the field's work is fixed by those two potentials alone, the two-term total may be evaluated at the start, evaluated again at the finish, and the two evaluations set equal. Substitute q and both potentials *with their signs* — the second one is negative here — then solve the single remaining unknown.
Annotated solution	Let the potential at A be V_A and the speed v_A, and at B be V_B and v_B. Take the ledger printed in the Point above and write it out with these numbers in place of the symbols: $\frac{1}{2}\times(6.6\times10^{-27})\times(2.0\times10^{5})^2 + (3.2\times10^{-19})\times(1.5\times10^{3}) = \frac{1}{2}\times(6.6\times10^{-27})\times v^2 + (3.2\times10^{-19})\times(-1.8\times10^{3})$ $v^2 = (2.0\times10^{5})^2 + 2\times(3.2\times10^{-19})\times\{(1.5\times10^{3}) - (-1.8\times10^{3})\}/(6.6\times10^{-27})$ $= 4.0\times10^{10} + 32\times10^{10} = 36\times10^{10}$ Therefore, $v =$ **6.0×10^{5} m/s**.

Reasonableness check	The potential drops by 3.3×10^{3} V, so the positive ion loses electrostatic potential energy and must speed up; the final 6.0×10^{5} m/s is larger than the starting 2.0×10^{5} m/s, as expected.
Explain it yourself	In one sentence, say why the ion gains speed even though the potential at B is negative, and why the sign of V_B must be kept.

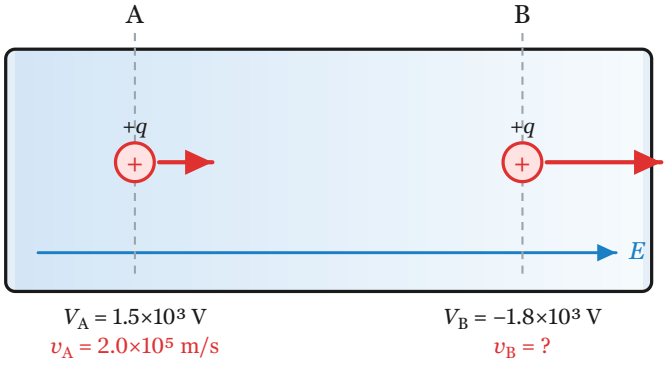


Figure 2-5-4 · The positive ion passes A with a short velocity arrow ($v_A = 2.0\times10^{5}$ m/s, $V_A = 1.5\times10^{3}$ V) and B with a longer one ($V_B = -1.8\times10^{3}$ V, $v_B = ?$).

✍ **Now you try:** A positive ion of mass 6.6×10^{-27} kg and charge 3.2×10^{-19} C is released from rest at point A (electric potential 1.65×10^{3} V) and moves to point B (electric potential 0 V) under the electrostatic force alone. Find its speed at B.

🔆 Try

1. A charged particle with a mass of 3.0×10^{-6} kg and a charge of 1.6×10^{-7} C was gently placed at point A (electric potential 8.0 V), and the particle started to move. Determine the speed in m/s when it passes point B (electric potential 2.0 V). Assume the particle is subjected only to the electrostatic force.

🌐 In a new context

An electron gun in a vacuum tube uses the same energy trade in reverse. An electron, which carries a negative charge, is released from rest near a heated filament and pulled toward a plate held at a higher potential. As it crosses the potential difference, its electrostatic potential energy falls and its kinetic energy rises, so it arrives at the plate moving fast — the principle behind the beam in an old television tube.

Figure 2-5-5 · Inside an evacuated tube, particles leave the hot filament almost at rest and are accelerated through the hole in the anode plate into a fast, straight beam that travels to the screen.

➤ **Your turn:** An electron (charge magnitude 1.6×10^{-19} C, mass 9.1×10^{-31} kg) starts from rest and is accelerated through a potential difference of 100 V. Using $\frac{1}{2}mv^2 = |q|V$, estimate its final speed.

🧪 Nature of Science

From the original — mechanism of a particle accelerator. In physics research, giant devices called "accelerators" are used to speed up electrons and protons to velocities close to the speed of light and make them collide. Inside, a strong electric field is created to pull the charged particles, and by converting electrostatic potential energy into kinetic energy the particles are driven to enormous speeds.

The single relation $\frac{1}{2}mv^2 + qV$ = constant, tested on a slow ion in a classroom, is the same energy bookkeeping that guides machines kilometers across — though at speeds approaching that of light the kinetic-energy term itself must

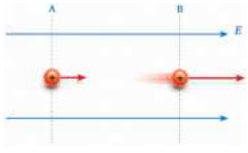
Figure 2-5-6 · A linear accelerator (linac): a charged particle travels in a straight line through a series of drift tubes, and an electric field in each gap gives it a push, converting electrostatic potential energy into kinetic energy so the particle keeps speeding up.

be replaced by its relativistic form. This is how physical models earn trust: a law confirmed in a simple, controlled case is applied far beyond it, and it keeps predicting correctly. The same energy accounting explains a marble on a ramp, an ion between two plates, and a proton in an accelerator.

J. J. Thomson turned this deflection into a measurement in 1897: from how far a beam of cathode rays bent in known fields he obtained the ratio of charge to mass, and the value lay so far from any atom's that he concluded he was looking at a new particle — the electron. Sometimes a ratio, measured carefully, is the discovery.

Your turn: Suggest one everyday device that speeds up charges by converting electrostatic potential energy into kinetic energy, and name the two energies being traded.

Lesson question, answered — Because the electrostatic force is conservative, a charge moving through an electric field conserves mechanical energy: $\frac{1}{2}mv^2 + qV$ = constant. As the charge moves to a potential where its electrostatic potential energy qV is lower, that energy becomes kinetic energy and the charge speeds up — exactly as a dropped ball trades gravitational potential energy for speed. The change in speed follows $v_B^2 = v_A^2 + 2q(V_A - V_B)/m$, with q and V signed.



The scene we opened with.

📋 Exercise

1. A positive ion with a mass of 6.6×10^{-27} kg and a charge of 3.2×10^{-19} C passed point A (electric potential 3.3×10^{3} V) at a speed of 2.0×10^{5} m/s. Determine the speed in m/s when this positive ion passes point B (electric potential 0 V). Assume the positive ion is subjected only to the electrostatic force.
2. A charge with a mass of m [kg] and a charge of q [C] ($q > 0$) that was stationary at point A (electric potential V_A [V]) received only an electrostatic force and passed point B (electric potential V_B [V]), where $V_B < V_A$. Determine the speed in m/s when it passed point B.

2-6 Capacitor

❓ **Lesson question:** What mechanism can temporarily store electricity and then release it as a current when it is needed?

You will learn to

1. Relate the stored charge to the potential difference across a capacitor, $Q = CV$, and give the capacitance C in farads (F).
2. Convert capacitance values between farads, microfarads (μF), and picofarads (pF).
3. Predict how halving or doubling the plate spacing changes the charge and the voltage, depending on whether the battery stays connected or is disconnected.

🔍 The phenomenon — predict before you continue

★ **Key idea**
Capacitance C [F] — the charge Q a capacitor holds at a given voltage V. How do Q and V go together?

A defibrillator sits silent until it is needed. At the press of a button a battery slowly charges a capacitor inside it up to a high voltage, storing a large charge $Q = CV$ on its plates. When the paddles fire, that stored charge floods out all at once — a single powerful pulse of current delivered in a fraction of a second. The device banks electricity quietly, then hands it all back on demand.

Figure 2-6-1 · A defibrillator charges a capacitor to a high voltage, storing charge $Q = CV$, then releases it as one brief, powerful pulse of current.

Predict

(a) As the capacitor charges up, is the charging current largest at the very first instant or once it is nearly full?

(b) Once the capacitor is charged and the battery is removed, does the stored charge stay on it or drain away at once?

🔍 Explore — Charge a capacitor, run by run

What you'll do: charge one capacitor to different voltages, release each charge through a bulb, and see how the stored charge tracks the charging voltage.

Time: 15 min

You will need:

- a capacitor
- a battery (adjustable voltage)
- a two-way switch
- a small bulb (to show the discharge)
- a coulombmeter to read the stored charge Q

capacitor

Battery

two-way switch

bulb

Steps

1. Set the two-way switch to the battery side and charge the capacitor to a chosen voltage V; read V across the plates.
2. Flip the switch to the bulb side; the capacitor discharges and the bulb flashes. Read the stored charge Q on the coulombmeter, and watch the flash as well — a brighter, longer flash goes with a larger Q.
3. Repeat at higher charging voltages, and for each run work out the ratio Q / V and enter it in the last column.
4. As a check, swap in a capacitor of larger capacitance and charge it to the same voltage.

Run	Charging voltage V (V)	Charge stored Q (μC)	Q / V (μF)
1			
2			
3			

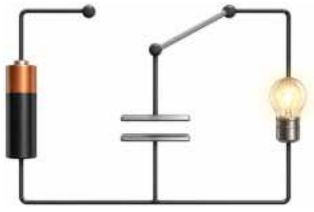


Figure 2-6-2 · A two-way switch connects the capacitor to the battery (charge) or to the bulb (discharge).

★ **Working note**

Let the charging voltage settle before you flip the switch, use the same bulb every run so equal flashes mean equal charge, and trust the trend in Q/V across the runs more than any single flash.

Analyze what you collected:

(a) As you raise the charging voltage V, what happens to the stored charge Q? Look down the last column — what do you notice about the ratio Q / V?

(b) A steady Q / V is the capacitance C. Use it to predict the charge this capacitor would store at a voltage you did not test, then say why a larger capacitor gave a bigger flash at the same V.

✎ Write your explanation in two lines before you read the next page.

From observation to model

Why the store fills in step with the voltage

Watch what the battery actually does. It drags electrons off one plate and piles them onto the other, and every electron it moves makes the next one harder to move — the emptied plate pulls back, the crowded plate pushes back. The transfer therefore does not run away; it stalls at the exact moment the plates have built a potential difference of their own equal to the battery's. Push twice as hard and the stall comes twice as late, with twice as much moved. That is where the straight-line relation in the Point comes from: it is not a law of nature discovered by experiment so much as

the arithmetic of a tug-of-war that always ends in a draw. And the constant of proportionality answers a single question — how much charge does this particular pair of plates swallow before it pushes back by one volt?

What the geometry decides

Two knobs control that number, and both act through the same chain of reasoning. Widen the plates and the same quantity of charge spreads more thinly; a thinner spread pushes back less, so the stall arrives later and more charge fits before the voltage is used up — the store grows with area. Now narrow the gap instead. The crowding on the plates is untouched, so the push-back per meter is untouched, but the charge has fewer meters to push across; a smaller voltage builds from the same charge, and again more charge fits before the battery is matched — the store grows as the gap shrinks. Halve the separation and you double what the device holds, which is exactly the change the Worked Example below puts to work.

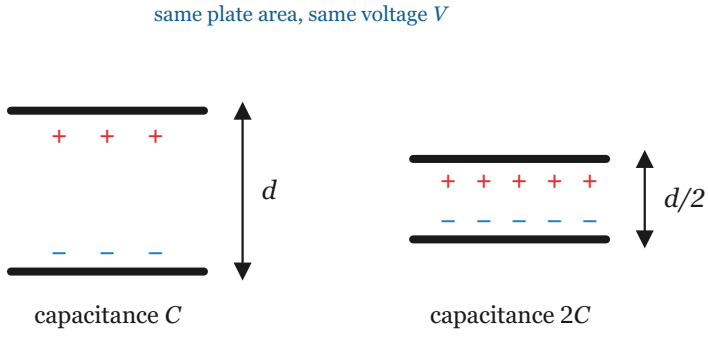
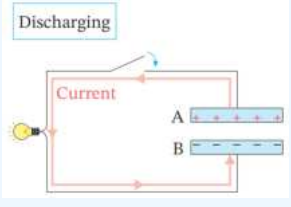
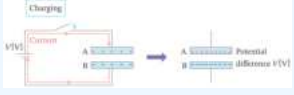


Figure 2-6-3 · Halving the plate spacing doubles the capacitance, so the closer plates store twice the charge at the same voltage.

Point





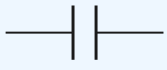


Figure 2-6-4 · The circuit symbol for a capacitor: two parallel plates.

When two metal plates of equal area face each other and are connected to the positive and negative terminals of a battery, respectively, electric charge moves as current flows, and charge is stored in the metal plates. Such a device is called a **capacitor**. These metal plates are called **electrodes** (or **plates**). The process of storing charge is called **charging**, and the process in which the stored charge flows out as a current is called **discharging**.

The charge Q [C] stored in a capacitor is proportional to the potential difference V [V] between the plates.

$$Q = CV$$

The proportionality constant C is called the **capacitance** of the capacitor, and its unit is the **farad** (symbol **F**).
Since the actual capacitance of a capacitor is often very small, 10^{-6} F is often used as 1 **microfarad** (symbol **μF**), and 10^{-12} F as a **picofarad** (symbol **pF**).
Example: 30 μF = **30×10^{-6} F** 45 pF = **45×10^{-12} F**

The capacitance of a capacitor is proportional to the **area** of the plates, and inversely proportional to the **spacing** of the plates.

★ **Term**

Capacitance is a property of the *hardware*, not of the charge sitting on it. Empty the device and C is unchanged; the number is set by the plates' shape, their separation, and what fills the gap. That is also where the model gives out: force enough charge on and the insulation between the plates breaks down, at which point the store dumps itself and no constant C describes anything.

★ **Key fact**

The farad is defined by 1 F = 1 C/V. A charge of 1 coulomb on a 1 farad capacitor sits at 1 volt — which is why the unit is almost never used undivided: a coulomb is an enormous quantity of charge to hold on a pair of plates, and ordinary components are millions or trillions of times smaller. The prefixes in the Point are not a convenience, they are a necessity.

📊 Equation box	
Capacitor charge	$Q = CV$ (unit: Q in C, C in F, V in V)
Unit prefixes	$1\ \mu F = 10^{-6}\ F$ $1\ pF = 10^{-12}\ F$
Geometry	$C \propto$ area / spacing $\Longrightarrow$ halving the spacing gives $C' = 2C$
State change	battery off: Q fixed, $V' = \frac{Q}{C'}$ battery on: V fixed, $Q' = C'V$

🛠 Worked Example

Situation	A parallel-plate capacitor with a capacitance of 4.0 μF and a vacuum between the plates was charged with a 6.0 V battery. (a) With the battery disconnected, the spacing between the plates was halved. Determine the potential difference V' [V] across the plates at this time. (b) With the battery connected, the spacing between the plates was halved. Determine the charge Q' [C] of the capacitor at this time.
Given and required	$C = 4.0\ \mu F = 4.0 \times 10^{-6}$ F, $V = 6.0$ V. (a) battery disconnected, spacing halved; find V'. (b) battery connected, spacing halved; find Q'.
Representation	See Figure 2-6-3 for how halving the spacing changes the capacitance. Sketch the plates before and after the spacing is halved (Figure 2-6-5).
Strategy	Check the capacitance C and the potential difference V, and substitute them into the basic formula $Q = CV$ to find the stored charge. Halving the spacing doubles the capacitance, $C' = 2C$. When the battery is disconnected the charge is fixed, so use $V' = Q/C'$; when the battery stays connected the voltage is fixed, so use $Q' = C'V$.

Annotated solution	Charge stored after charging: $Q = CV = (4.0\times10^{-6})\times6.0 =$ **2.4×10^{-5} C**. With the spacing halved, $C' = 2C = 2\times(4.0\times10^{-6}) = 8.0\times10^{-6}$ F. **(a)** With the battery disconnected, the charge Q is kept constant. From $Q = C'V'$, $V' = Q/C'$ $V' = (2.4\times10^{-5}) / (8.0\times10^{-6}) =$ **3.0 V**. **(b)** With the battery connected, the potential difference between the plates is kept constant at 6.0 V. $Q' = C'V = (8.0\times10^{-6})\times6.0 =$ **4.8×10^{-5} C**.
Reasonableness check	(a) The charge is unchanged but the capacitance has doubled, so $V = Q/C$ must halve: 6.0 V → 3.0 V, as found. (b) The voltage is unchanged but the capacitance has doubled, so the charge must double: 2.4×10^{-5} C → 4.8×10^{-5} C, as found.
Explain it yourself	In one sentence, say why disconnecting the battery keeps Q fixed while leaving the battery connected keeps V fixed.

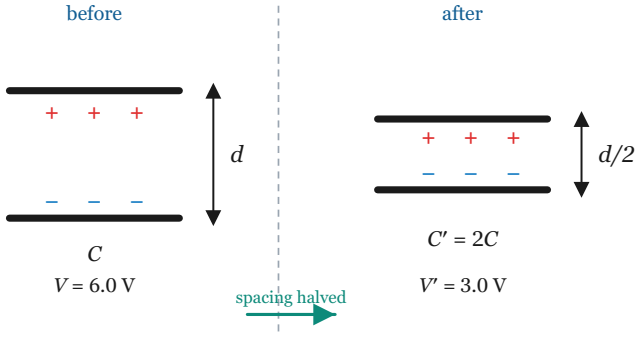


Figure 2-6-5 · The same capacitor before and after the spacing is halved: the gap becomes d/2 and the capacitance doubles to C′ = 2C. Drawn for part (a), with the battery disconnected: the charge on the plates is unchanged, so the potential difference halves to 3.0 V.

✍ **Now you try:** A 2.0 µF parallel-plate capacitor with a vacuum between the plates is charged by a 12 V battery. With the battery disconnected, the spacing is halved. Find the new potential difference.

Try

1. Determine the following to 2 significant figures. When a voltage of 30 V is applied to a capacitor with a capacitance of 6.0 µF, determine the charge of the capacitor in C.
2. A parallel-plate capacitor with a capacitance of 20 pF and a vacuum between the plates was charged with a 15 V battery.
   (i) With the battery disconnected, the spacing between the plates was doubled. Determine the charge Q_1 [C] of the capacitor and the potential difference V_1 [V] between the plates at this time.
   (ii) With the battery connected, the spacing between the plates was doubled. Determine the charge Q_2 [C] of the capacitor and the potential difference V_2 [V] between the plates at this time.

★ **Note**

Battery disconnected → charge Q is fixed, so $V = Q/C$. Battery connected → voltage V is fixed, so $Q = CV$. A capacitor storing charge Q holds $+Q$ on one plate and $-Q$ on the other, so it stays neutral overall. Q is the charge *moved* from one plate to the other, not charge created.

🌐 In a new context

A camera's electronic flash relies on a capacitor. Between shots a battery slowly charges the capacitor to a high voltage, storing charge $Q = CV$ on its plates. When the shutter fires, the capacitor discharges through the flash tube in a tiny fraction of a second, delivering all that stored charge as a brief, intense burst of light that a small battery could never supply on its own. The same store-then-release idea powers the defibrillators used to restart a heart.

➤ **Your turn:** A camera flash uses a 100 μF capacitor charged to 300 V. Using $Q = CV$, find the charge it stores.

Figure 2-6-6 · A small battery slowly charges a large capacitor, which then dumps its stored energy in a single sudden burst through the flash tube to fire the camera's flash.

🧪 Nature of Science

From the original — smartphone touch panels. You can operate the screen of a smartphone or tablet by touching it because it uses a capacitor mechanism called the capacitive-touch method. A weak electric field sits on the surface of the screen, and when a finger touches it a tiny capacitor forms between the finger and the screen, changing the amount of stored charge, that is, the capacitance. A sensor reads this change to identify the touched position.

This is a good example of how physics turns an invisible quantity into a measurement: the finger changes a capacitance far too small to feel, yet the device detects that change reliably and converts it into a position on the screen. A quantity is useful in science when a clear procedure can measure it, not only when it can be seen.

The first store of this kind was found twice over: by von Kleist in October 1745 and by Musschenbroek at Leiden a few months later, neither aware of the other. Both took the electricity to be held in the water inside the jar, and Franklin's tests showed it sat in the glass. Independent discovery is common in physics, and the first explanation of a new effect is often the wrong one.

Your turn: Suggest one other everyday device that works by detecting a change in stored charge, and name the quantity it effectively measures.

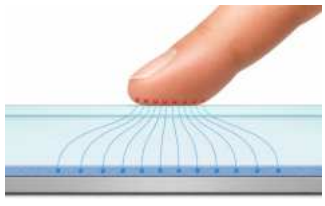


Figure 2-6-7 · Touching the screen forms a tiny capacitor between fingertip and surface; the sensor reads the change in stored charge as a touch position.

Lesson question, answered — A capacitor stores electricity: two facing electrodes hold a charge $Q = CV$ that stays put when the battery is removed, and it releases that charge as a current when the plates are connected through a circuit (discharging). The capacitance C, in farads, is fixed by the plate area and spacing, so the same device can be tuned to store as much charge as a task needs.

The scene we opened with.

📋 Exercise

1. Determine the following to 2 significant figures. When a voltage of 15 V is applied to a capacitor with a capacitance of 3.0 μF, determine the charge of the capacitor in C.
2. Determine the following to 2 significant figures. When a voltage of 60 V is applied to a capacitor with a capacitance of 5.0 pF, determine the charge of the capacitor in C.

3. Determine the following to 2 significant figures. A parallel-plate capacitor with a capacitance of 3.0 μF and a vacuum between the plates was charged with a 20 V battery.
   (i) With the battery disconnected, the spacing between the plates was halved. Determine the charge Q_1 [C] of the capacitor and the potential difference V_1 [V] between the plates at this time.
   (ii) With the battery connected, the spacing between the plates was halved. Determine the charge Q_2 [C] of the capacitor and the potential difference V_2 [V] between the plates at this time.
4. Determine the following to 2 significant figures. A parallel-plate capacitor with a capacitance of 40 pF and a vacuum between the plates was charged with a 20 V battery.
   (i) With the battery disconnected, the spacing between the plates was doubled. Determine the charge Q_1 [C] of the capacitor and the potential difference V_1 [V] between the plates at this time.
   (ii) With the battery connected, the spacing between the plates was doubled. Determine the charge Q_2 [C] of the capacitor and the potential difference V_2 [V] between the plates at this time.

UNIT 3

Electric Current and Circuits

3-1	Ammeter and Voltmeter	88
3-2	Kirchhoff's Laws	95

3-1 Ammeter and Voltmeter

❓ **Lesson question:** How can an ammeter or voltmeter measure currents and voltages far beyond its own measurement range?

You will learn to

1. Connect an ammeter in series and a voltmeter in parallel, and explain why an ammeter needs a small internal resistance while a voltmeter needs a large one.
2. Extend the range of a galvanometer with a shunt in parallel, using the equal-voltage condition and Ohm's law to find the shunt resistance.
3. Extend the range of a galvanometer with a multiplier in series, using the equal-current condition and Ohm's law to find the multiplier resistance.

🔍 The phenomenon — predict before you continue

A small galvanometer movement is marked to read at most 10 mA, yet you need to measure a current of 200 mA in a circuit. Sending the whole current straight through the movement would drive the needle far past its limit and could burn out the coil. Still, real workshop meters read currents and voltages hundreds of times larger than the bare movement allows — without being destroyed.

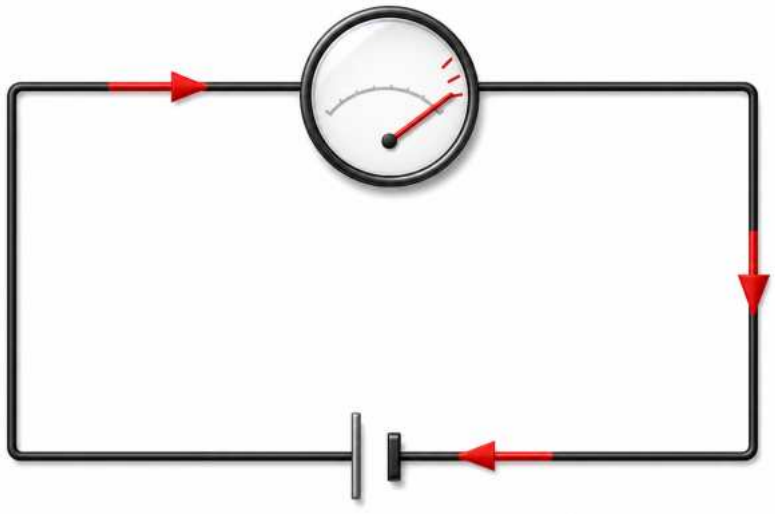


Figure 3-1-1 · A single bare galvanometer movement rated for only 10 mA is placed directly in a branch carrying 200 mA, so its needle is driven far past full scale.

Predict

(a) To let a bare 10 mA movement register a 200 mA line current, where would you route the extra 190 mA, and how would you connect that path — in series or in parallel with the movement?

(b) To read a voltage larger than a voltmeter's limit, would you add a resistor in series or in parallel with the meter?

★ **Key idea**

An ammeter is dropped into the path; a voltmeter is bridged across it. Each carries an internal resistance of its own. What must that resistance be if the meter is to report the circuit rather than rewrite it — and how can a movement be made to read beyond its own full scale?

🔍 Explore — Shunt a milliammeter to widen its range

What you'll do: Wire different low-value resistors across a milliammeter and see how much wider a line current it can then read, uncovering the current-division ratio a shunt sets.

Time: 15 min

You will need:

- A milliammeter movement whose full-scale reading is known
- A reference ammeter to read the true line current
- A DC supply and a rheostat to set the current safely
- A set of small low-value resistors to try as shunts

ammeter

DC supply

rheostat

shunt resistor

Steps

1. With no shunt fitted, raise the supply until the milliammeter reads full scale, and record that current as the meter reading I_g.
2. Wire a small resistor directly across the two meter terminals, then trim the rheostat until the milliammeter again sits at full scale.
3. Read the line current I on the reference ammeter, and work out the multiplication factor $n = I / I_g$.
4. Swap in a smaller shunt resistor and repeat.

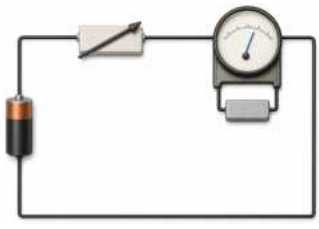


Figure 3-1-2 · Bench setup: a milliammeter in series with a supply and a rheostat, with a small resistor wired directly across the meter terminals so part of the line current bypasses the movement.

Run	Meter reading I_g (mA)	Line current I (mA)	Factor $n = I / I_g$
1			
2			
3			

> ★ **Working note**
> Read the milliammeter and the reference ammeter at the same instant, and keep the movement at or below its full-scale mark by adjusting the rheostat — never the shunt — so the coil is never overdriven.

Analyze what you collected:

(a) As you moved to a smaller shunt resistance, what happened to the factor n, and so to the share of the line current that bypassed the movement?

(b) The meter and shunt share the same two terminals at equal voltage. Use that to predict which shunt you would fit to make the movement read a line current 20 times its own full scale.

✎ Write your explanation in two lines before you read the next page.

From observation to model

Why a meter must be almost invisible

An instrument is meant to report a circuit, not rewrite it — and each of these two rewrites it by exactly the amount of resistance it carries. Break a branch open and drop a meter into the gap, and you have added a resistance to the very branch whose current you came to find; what the needle then shows is already less than what flowed before you arrived, short by a fraction set by how the meter's resistance compares with everything else in that branch. Drive that resistance toward nothing

and the damage goes with it. Bridging a meter across a component commits the opposite offense: you have handed the current a second road to travel, and whatever takes it never reaches the component, whose voltage sags accordingly. The cure is now the opposite extreme — make the second road so punishing that almost nothing chooses it. The two rules in the Point are therefore not two facts to memorize but one idea seen from both ends: a meter earns its reading by stealing as little as it can of the quantity it is *not* measuring.

Trading range for reading

Neither cure widens the range by itself; the movement still burns out above its own limit. Widening means sending the surplus elsewhere and reporting only the fixed share left behind. Fit a low resistance beside the ammeter and the line divides at that junction. Both roads begin and end at the same pair of terminals, so both must stand at the same voltage — and Ohm's law then splits the current between them in the *inverse* ratio of their resistances. Give the bypass one-ninth of the movement's resistance and it carries nine times the movement's current, so ten parts arrive and the needle is shown one. Put a large resistance in front of the voltmeter and the mirror image runs: one road, therefore one current, therefore a voltage shared in *direct* ratio to the resistances, the surplus dropped harmlessly outside the movement. Each design condition is nothing more than the definition of its connection, and Ohm's law turns the definition into a number.

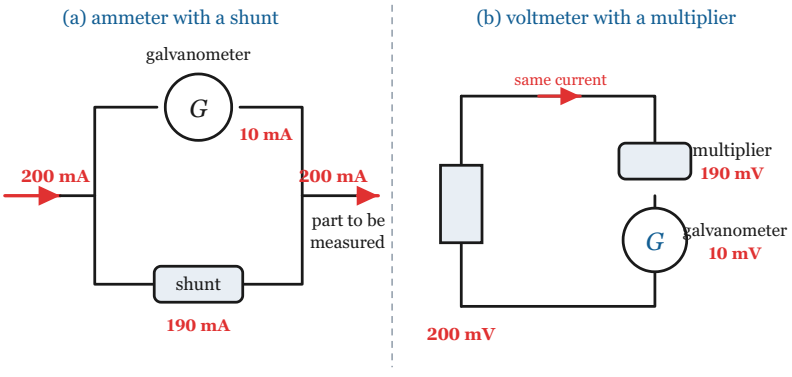


Figure 3-1-3 · The two arrangements set side by side, so the mirror symmetry between them can be read off in one glance: on the left the added resistor sits *beside* the movement, which forces the two to share a voltage and lets them divide a current; on the right it sits *ahead* of the movement, which forces them to share a current and lets them divide a voltage. Each design condition is simply the property of its own connection — that is the reasoning the Point's two figures leave implicit.

Point

! To measure the current flowing in a certain part of a circuit, an ammeter is connected in **series** .
An ammeter has an internal resistance. To minimize its effect on the current flowing in the part to be measured, the internal resistance should be as **small** as possible.

! To expand the measurement range of a galvanometer, a resistor is connected in **parallel** with the galvanometer to allow a portion of the current to bypass it. This resistor is called a **shunt** of the galvanometer.
Example: When you want to measure a current up to 200 mA with a galvanometer that can measure up to 10 mA, if 10 mA of the current flows into the galvanometer and the remaining 190 mA is allowed to bypass through the shunt, the total current is 20 times the value indicated by the galvanometer.

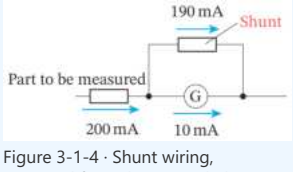


Figure 3-1-4 · Shunt wiring, imported from the original: the meter dial and a low resistance bridge one and the same pair of junctions, so the line arriving from the left divides between the two branches and reunites on the right. The arrow labels give the division for the Example beside it.

★ **Term**

Shunt — the bypass road, fitted beside the movement. Note what it costs: the meter now reports only a fixed share of the line, so every reading must be multiplied, and the accuracy of the answer is only as good as the accuracy of that low resistance. A shunt whose value drifts with heat quietly falsifies every reading taken through it.
Multiplier — the voltage-absorbing partner, fitted ahead of the movement. It buys range at the price of drawing an ever smaller current from the circuit under test, which is welcome, and of putting a large, temperature-sensitive resistance in the reading path, which is not.

★ **Key fact**

The widening factor follows from the resistance ratio alone. With a bypass of resistance R_s beside a movement of internal resistance R_g, the line current is $(1 + R_g/R_s)$ times what the needle shows — so a factor of 20 needs $R_s = R_g/19$. In front of a voltmeter the ratio inverts: the applied voltage is $(1 + R_m/R_g)$ times the reading, so the same factor of 20 needs $R_m = 19R_g$.

! To measure the voltage in a certain part of a circuit, a voltmeter is connected in **parallel** .
A voltmeter has an internal resistance. Since it is connected in parallel with the part to be measured, the current flowing through the voltmeter should be as small as possible. For that purpose, the internal resistance should be as **large** as possible.

! To expand the measurement range of a galvanometer, a resistor is connected in **series** with the galvanometer to drop a portion of the voltage. This resistor is called a **multiplier** of the galvanometer.
Example: When you want to measure a voltage up to 200 mV with a galvanometer that can measure up to 10 mV, if 10 mV of the voltage is applied to the galvanometer and the remaining 190 mV is dropped across the multiplier, the total voltage is 20 times the value indicated by the galvanometer.

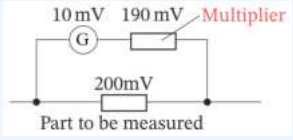


Figure 3-1-5 · Multiplier wiring, imported from the original: dial and resistor lie one behind the other along a single branch, itself bridged across the component under test, so one current threads both and the labels show how the applied 200 mV is shared out.

Equation box

Ammeter and shunt	in parallel, voltages equal: $R_g I_g = R_s I_s$, with $I_s = I - I_g$ (I_g through the movement, I_s through the shunt, I the line current)
Voltmeter and multiplier	in series, currents equal: $\frac{V_g}{R_g} = \frac{V_m}{R_m}$, with $V_m = V - V_g$ (V_g across the movement, V_m across the multiplier, V the applied voltage)
Ohm's law (prior)	$V = IR$

Worked Example

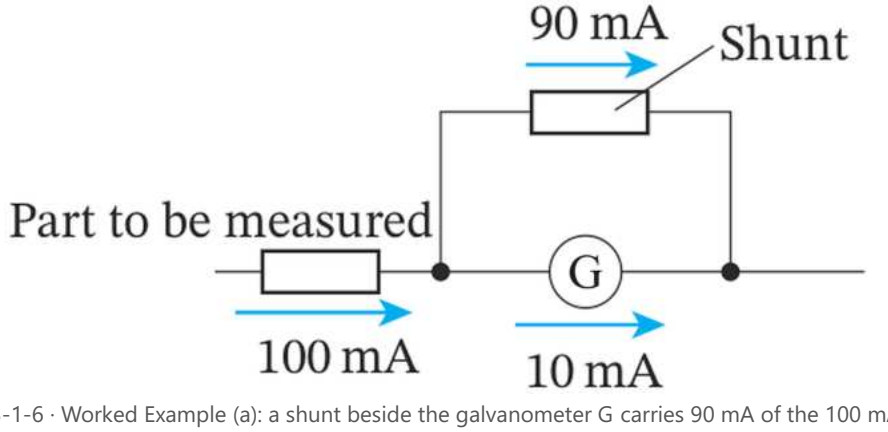


Figure 3-1-6 · Worked Example (a): a shunt beside the galvanometer G carries 90 mA of the 100 mA line current, so the needle still reads only its 10 mA full-scale value.

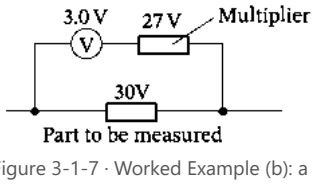


Figure 3-1-7 · Worked Example (b): a multiplier in series with the galvanometer takes 27 V of the 30 V across the part being measured, leaving 3.0 V across the meter itself.

Situation	Determine the following. (a) A galvanometer with an internal resistance of 1.8 Ω can measure up to 10 mA. To measure currents up to 100 mA using this, determine the required resistance of the shunt in Ω. (b) There is a galvanometer with an internal resistance of 3.0 kΩ that can measure voltages up to 3.0 V. To measure voltages up to 30 V using this, determine the required resistance of the multiplier in kΩ.
Given and required	(a) internal resistance 1.8 Ω, meter full scale 10 mA, target range 100 mA; find shunt resistance R_s [Ω]. (b) internal resistance 3.0 kΩ, meter full scale 3.0 V, target range 30 V; find multiplier resistance R_m [kΩ].
Representation	The figure above the table is part (a): the galvanometer and the shunt in parallel, with the 100 mA line splitting 10 mA / 90 mA between them. Figure 3-1-7 is part (b): the galvanometer and the multiplier in series, sharing one current.
Strategy	A shunt that expands the range of a galvanometer is connected in parallel, so set up an equation using Ohm's law from the condition that the voltage across meter and shunt is equal. A multiplier that expands the range of a galvanometer is connected in series, so set up an equation from the condition that the current through meter and multiplier is equal.
Annotated solution	**(a)** Since the galvanometer and the shunt are in parallel, the applied voltages are equal. When a current of 100 mA flows through the part to be measured, 10 mA should flow through the galvanometer and 90 mA through the shunt. Let the shunt resistance be R_s [Ω]. From Ohm's law, 1.8 Ω × 10 mA = R_s [Ω] × 90 mA Therefore, R_s = 18/90 = **0.20 Ω**. **(b)** Since the galvanometer and the multiplier are in series, the currents flowing through them are equal. When a voltage of 30 V is applied to the part to be measured, 3.0 V should be applied to the galvanometer and 27 V to the multiplier. Let the multiplier resistance be R_m [kΩ]. From Ohm's law, 3.0 V / 3.0 kΩ = 27 V / R_m [kΩ] Therefore, R_m = 27 × 3.0 / 3.0 = **27 kΩ**.

Reasonableness check	(a) The shunt must carry 9 times the meter current at the same voltage, so its resistance should be one-ninth of 1.8 Ω, and 1.8/9 = 0.20 Ω fits. (b) The multiplier must drop 9 times the meter voltage at the same current, so its resistance should be 9 times 3.0 kΩ, and 9 × 3.0 = 27 kΩ fits.
😳 **Explain it yourself**	In one sentence, say why the shunt equation uses equal voltage while the multiplier equation uses equal current.

✍️ **Now you try:** A galvanometer with an internal resistance of 4.0 Ω reads up to 5.0 mA. Find the shunt resistance needed to read up to 25 mA.

🔆 Try

1. There is a galvanometer with an internal resistance of 3.0 Ω that can measure up to 1.0 A. To measure currents up to 3.0 A using this, determine the required resistance of the shunt in Ω.
2. There is a galvanometer with an internal resistance of 3.0 kΩ that can measure voltages up to 5.0 V. To measure voltages up to 30 V using this, determine the required resistance of the multiplier in kΩ.

🌐 In a new context

A modern digital multimeter hides this same idea inside its rotary switch. Turning the dial to a higher current range simply connects a smaller shunt across the same internal movement, so more of the line current bypasses it; turning to a higher voltage range switches in a larger multiplier in series, so more of the applied voltage is dropped outside the movement. One tiny sensor is reused across ranges spanning many powers of ten purely by selecting resistors.

➤ **Your turn:** A meter movement reads up to 50 mV across itself. What multiplier resistance, in series, lets it read up to 500 mV if the movement draws 1.0 mA at full scale?

Figure 3-1-8 · A single central meter movement is reused for every range, with short thick low-resistance shunts wired in parallel for current ranges and long thin high-resistance multipliers wired in series for voltage ranges, one at a time selected by the rotary switch.

🧪 Nature of Science

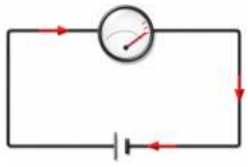
From the original — internal resistance of ammeters and voltmeters. An ideal ammeter has an internal resistance of 0 Ω, and an ideal voltmeter has an infinite internal resistance. This is to ensure that connecting an ammeter in series does not increase the overall resistance of the circuit, and connecting a voltmeter in parallel prevents unnecessary current from escaping into it. Actual meters are also designed to approach these ideals.

The ideal ammeter and ideal voltmeter are limiting models: no real meter reaches 0 Ω or infinite resistance, yet engineers push each design toward its ideal until the leftover disturbance is small enough to ignore for the measurement at hand. A good measuring instrument is one that reports a quantity while changing it as little as possible.

The movement at the heart of these instruments is the moving coil of d'Arsonval and Deprez, built in 1882, which Weston made portable and accurate enough to sell in 1888. Progress in physics is often progress in instruments: a quantity becomes ordinary only once something reliable can read it.

Your turn: Suggest one measurement in which a voltmeter's finite internal resistance would noticeably change the reading, and say which circuit feature makes the effect large.

Lesson question, answered — The method is to keep only a small, fixed fraction of the quantity inside the meter movement and divert the rest through an added resistor. For large currents, a low-resistance shunt in parallel carries the excess current, sized by equal voltage; for large voltages, a high-resistance multiplier in series drops the excess voltage, sized by equal current. In both cases Ohm's law fixes the added resistance.



The scene we opened with.

📋 Exercise

1. There is a galvanometer with an internal resistance of 9.0 Ω that can measure up to 30 mA. To measure currents up to 300 mA using this, determine the required resistance of the shunt in Ω.
2. There is a galvanometer with an internal resistance of 2.0 kΩ that can measure voltages up to 5.0 V. To measure voltages up to 50 V using this, determine the required resistance of the multiplier in kΩ.
3. Fill in the blanks in the following sentences. To measure the current flowing in a certain part of a circuit, an ammeter is connected in (i). The internal resistance of the ammeter should be as (ii) as possible. To measure the voltage in a certain part of a circuit, a voltmeter is connected in (iii). The internal resistance of the voltmeter should be as (iv) as possible.

3-2 Kirchhoff's Laws

❓ **Lesson question:** In a circuit where the current branches in complex ways, what regularities hold among the currents and voltages in each path?

You will learn to

1. Tell a resistor's descent in potential apart from a source's lift, and calculate each from the quantities already given.
2. Write a junction equation for any meeting point of branches, using Kirchhoff's first law.
3. Write a loop equation for any closed path, using Kirchhoff's second law, and solve the set of simultaneous equations for every branch current — reading a negative answer as a reversed arrow.

🔍 The phenomenon — predict before you continue

Two batteries of different voltages are wired into the same set of resistors, and at one junction three wires meet, so the current can split and rejoin. A single resistor is shared by both loops. Before solving anything, look at the branch that holds the smaller battery.

★ **Key idea**
Each wire meeting at a junction carries its own branch current I [A], and you choose a direction for each before you can add them.

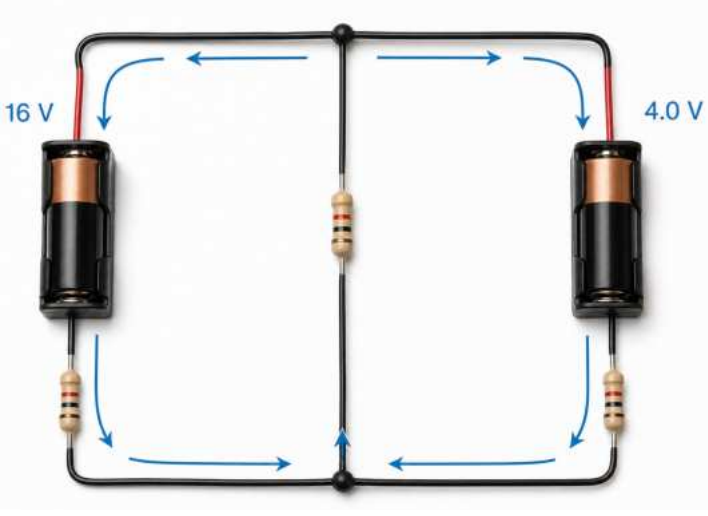


Figure 3-2-1 · Two batteries of unequal emf drive three resistors in two loops that share a middle branch and meet at a junction.

Predict

(a) In which direction does the current flow in the branch that holds the smaller battery — the same way as the larger battery drives it, or against it?

(b) Can the current in one branch turn out to run opposite to the direction you first assumed?

🔍 Explore — Test the junction rule in a two-loop circuit

What you'll do: Measure the three branch currents where two loops meet and check whether the current flowing into the junction really equals the current flowing out.

Time: 20 min

You will need:

- Two cells of different emf (e.g. 9 V and 4.5 V)
- Three resistors
- An ammeter (one moved between branches, or three)
- A voltmeter and connecting wires

Two cells

Resistors

Ammeter

Voltmeter

Steps

1. Build the two-loop circuit and draw an assumed arrow for the current in each of the three branches: I_1 and I_2 in the two outer branches, I_3 in the shared middle branch.
2. Place the ammeter in the first outer branch and read I_1; move it to the second outer branch for I_2, then to the middle branch for I_3.
3. For each run compute the derived value $I_1 + I_2 - I_3$ and enter it in the last column.
4. (Optional) With the voltmeter, measure the drop across each resistor and each cell's emf, and add them around one closed loop.

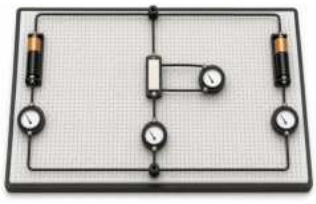


Figure 3-2-2 · A two-loop test circuit: two cells in the outer branches, a shared middle resistor, an ammeter (A) in each branch, and a voltmeter (V) across the resistor.

Run	I_1 in (A)	I_2 in (A)	I_3 out (A)	$I_1 + I_2 - I_3$ (A)
1				
2				
3				

Analyze what you collected:

(a) Look down the last column: what single value does $I_1 + I_2 - I_3$ stay close to across every run, and what does that tell you about how much charge collects at the junction?

(b) If one branch current came out negative, what does its sign mean, and does the junction balance still hold when you keep that sign? Predict what your voltmeter loop-sum should equal.

✎ Write your explanation in two lines before you read the next page.

★ **Working note**

Always connect the ammeter in series inside a branch and the voltmeter in parallel across a component. Note the sign your meter shows: if a branch reads negative, the real current runs opposite to your assumed arrow — record it with its sign, do not flip the leads.

From observation to model

Voltage drop and electromotive force

Think of potential as height and the circuit as a hilly path. A source is the only thing that lifts charge; every resistor lets it back down. The size of each descent is not a new quantity to be looked up — Ohm's law already fixes it, since the same R and the same I that define a resistor's behavior also fix how far the potential must fall across it. That is the whole content of the first term named in the Point: a descent,

measured in volts, whose size you can already calculate. The lifting term is its counterpart, and it is worth being careful about what it measures: not a force at all, despite its name, but the energy each coulomb is handed on its way through the source. For an ideal source — one with no resistance of its own to descend through — that handover shows up undiminished at the terminals, which is why it can be read there with a voltmeter.

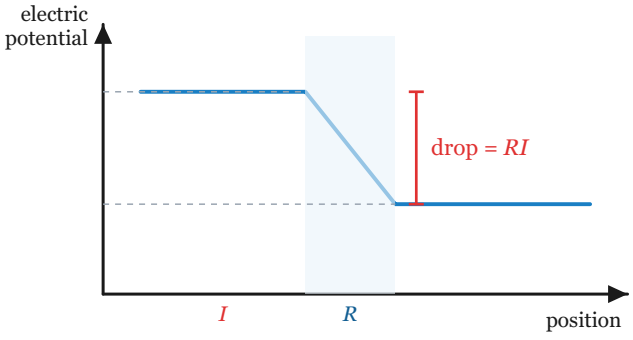


Figure 3-2-3 · Electric potential along a conductor: it holds steady, then steps down by the voltage drop *RI* across the resistor of resistance *R* carrying current *I*.

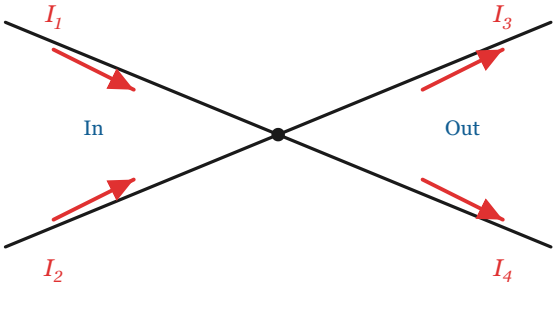
> ★ **Key fact**
> Both quantities are measured in volts, and both reduce to the same unit identity: 1 V = 1 Ω·A = 1 J/C. A resistor of 20 Ω passing 0.50 A drops 10 V — ten joules from every coulomb that crosses it, converted to heat.

> ★ **Term**
> **Electromotive force** — a name inherited from the nineteenth century and misleading ever since: the quantity is an energy per unit charge, measured in volts, not a force in newtons. Note also the limit hidden in the ideal case above. A real cell carries resistance inside itself, so part of its lift is spent internally and the terminals show less than the full figure — the more current drawn, the wider that gap. Every calculation in this lesson assumes the gap is zero.

Where the two rules come from

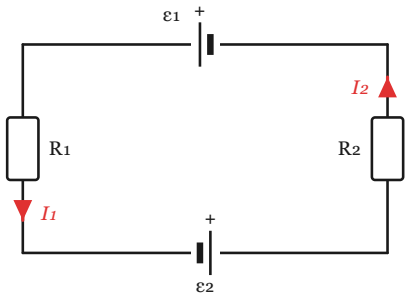
Neither rule is a new discovery about electricity; each is an old conservation law wearing circuit clothing. Charge cannot be created, destroyed, or stockpiled at a point of wire, so a meeting place of branches can only pass along what reaches it — whatever arrives per second must leave per second, which is the first rule and nothing more. Energy cannot be created or destroyed either, and potential is energy per coulomb; carry one coulomb all the way round a closed path and it must come back to the height it started from, so the lifting it received must be matched exactly by the descents it made. That is the second rule.

The procedure in the Point follows from this, and its opening instruction deserves attention. You are told to *choose* arrows before you know anything — and the choice cannot be wrong, only unlucky. Pick badly and the algebra hands back a minus sign, which is not an error message but an answer: the arrow was drawn backwards, the size is right. This is what makes the method mechanical rather than clever. You do not have to reason out where current goes; you assert, write the two kinds of equation, and let the signs correct you. Each junction supplies one equation, each independent loop supplies another, and once there are as many equations as unknown currents the circuit is solved — the two figures below show one junction and one loop, each with the equation it yields.



$$I_1 + I_2 = I_3 + I_4$$

Figure 3-2-4 · Kirchhoff's first law at a junction: the two currents flowing in equal the two flowing out, $I_1 + I_2 = I_3 + I_4$.



$\varepsilon_1 + \varepsilon_2 = R_1 I_1 + R_2 I_2$

Figure 3-2-5 · Kirchhoff's second law around a closed loop with two sources and two resistors: $\varepsilon_1 + \varepsilon_2 = R_1 I_1 + R_2 I_2$.

Point

- When a current I [A] flows through a conductor with a resistance of R [Ω], the decrease in potential by ***RI*** across the resistor is called a **voltage drop**.
- The potential difference between the positive and negative terminals of a battery or power supply is called **electromotive force**.
- When considering complex circuits, use **Kirchhoff's laws**.
  1. Assume the direction of the current.
  2. Kirchhoff's first law. Focus on the junctions in the circuit and set up equations.

     Sum of currents flowing in = **Sum of currents flowing out**
  3. Kirchhoff's second law. Focus on a closed loop in the circuit and set up equations.

     Sum of electromotive forces = **Sum of voltage drops**

Equation box

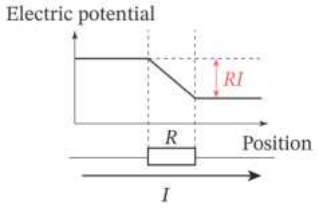
Voltage drop	$= RI$ (unit: V = Ω·A)
First law, in symbols	$\Sigma I_{in} = \Sigma I_{out} \implies$ for the junction drawn above, $I_1 + I_2 = I_3 + I_4$
Second law, in symbols	$\Sigma\varepsilon = \Sigma RI \implies$ for the loop drawn above, $\varepsilon_1 + \varepsilon_2 = R_1 I_1 + R_2 I_2$

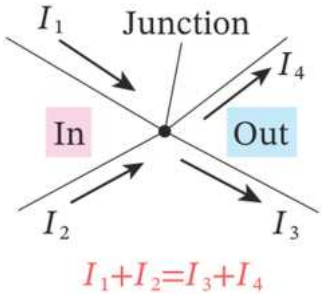
Worked Example

Situation	A circuit consists of a battery and resistors as shown in Figure 3-2-6: electromotive forces 16 V and 4.0 V, a 20 Ω resistor in path I, a 20 Ω resistor in the shared branch, and a 10 Ω resistor in path II, meeting at junction a. Determine the direction and magnitude of the current flowing through the 10 Ω resistor.

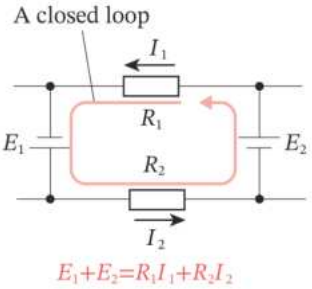
★ Note

Two traps in the bookkeeping. First, a loop equation is only true if you keep one consistent sense of travel all the way round it — reverse halfway and the descents you have already counted change sign under you. Second, count your equations before you start solving: three unknown currents need three independent relations, and re-using the same junction twice supplies no new information, only a rearrangement of the one you already had. Current is not used up as it goes round. The first law says the opposite: whatever flows into a junction flows out of it. What the resistors consume is *energy*, not charge.





$I_1 + I_2 = I_3 + I_4$



$E_1 + E_2 = R_1 I_1 + R_2 I_2$

Given and required	Electromotive forces ε = 16 V (path I) and 4.0 V (path II); resistances 20 Ω (path I), 20 Ω (shared), 10 Ω (path II). Find the direction and magnitude of the current in the 10 Ω resistor.
Representation	Let the currents through the resistors be I_1 (path I, 20 Ω), I_2 (path II, 10 Ω), and I_3 (shared, 20 Ω), with the directions assumed in Figure 3-2-6, all meeting at junction a.
Strategy	Step 1: assume a direction for each current. Step 2: apply Kirchhoff's first law at junction a. Step 3: apply Kirchhoff's second law to path I and to path II. Solve the three equations simultaneously.
Annotated solution	Using Kirchhoff's first law, for point a: $I_1 + I_2 = I_3$ .....(i) Using Kirchhoff's second law, for path I: $16 = 20\times I_1 + 20\times I_3$ .....(ii) for path II: $4.0 = 10\times I_2 + 20\times I_3$ .....(iii) Solving (i), (ii), and (iii) simultaneously gives **I_1 = 0.50 A**, **I_2 = −0.20 A**, and **I_3 = 0.30 A**. Since I_2 is negative, its direction is opposite to the assumed direction. Therefore, the current flowing through the 10 Ω resistor is directed **downward** (opposite to the assumed direction), with magnitude **0.20 A**.
Reasonableness check	Substitute $I_3 = I_1 + I_2$ into (ii) and (iii): (ii) becomes $40I_1 + 20I_2 = 16$, i.e. $2I_1 + I_2 = 0.80$; (iii) becomes $20I_1 + 30I_2 = 4.0$, i.e. $2I_1 + 3I_2 = 0.40$. Subtracting gives $2I_2 = -0.40$, so $I_2 = -0.20$ A, then $I_1 = 0.50$ A and $I_3 = 0.30$ A — matching the answers above. The negative I_2 is consistent with the smaller battery being overpowered by the larger one.
😳 **Explain it yourself**	In one sentence, say what a negative value of I_2 tells you physically, and why it does not mean a mistake was made.

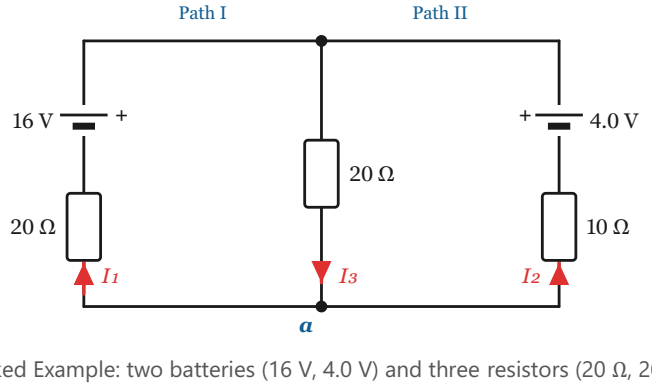


Figure 3-2-6 · Worked Example: two batteries (16 V, 4.0 V) and three resistors (20 Ω, 20 Ω, 10 Ω) form two loops sharing a middle branch that meet at junction *a*; assumed currents I_1, I_2, I_3.

✍ **Now you try:** Two batteries, 9.0 V and 6.0 V, drive current into a shared branch; each outer branch and the shared branch holds a 10 Ω resistor. Find the current in the shared resistor.

🔆 Try

1. A circuit consists of a battery and resistors as shown in Figure 3-2-7. Determine the direction and magnitude of the current flowing through each resistor.

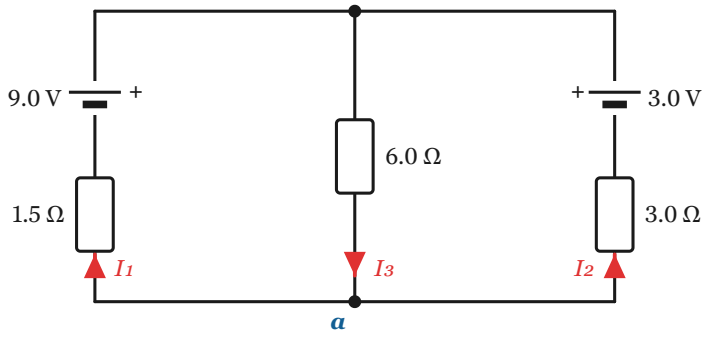


Figure 3-2-7 · Try: batteries 9.0 V and 3.0 V with resistors 1.5 Ω, 6.0 Ω, and 3.0 Ω in a two-loop network — the 9.0 V battery shares its branch with the 1.5 Ω resistor, the 3.0 V battery with the 3.0 Ω resistor, and the 6.0 Ω resistor is the shared middle branch.

🌐 In a new context

Kirchhoff's laws describe far more than electric circuits. His first law is like counting cars at an intersection: the number of cars entering equals the number leaving, so traffic is conserved just as current is conserved at a junction. His second law is like hiking a mountain trail: if you climb and descend and return to your starting point, your net change in altitude is zero — exactly as a charge's net change in potential around any closed loop is zero. With only these two rules you can analyze any road network, however tangled, and any circuit built the same way.

Figure 3-2-8 · Two everyday analogies for Kirchhoff's laws: at a road junction the stream of cars entering balances the stream leaving (junction rule), while a hilltop trail forms a closed loop back to its starting marker so the net change in height around the loop is zero (loop rule).

➤ **Your turn:** At an intersection, two roads bring in 400 and 250 cars per minute, and one exit road carries away 300 cars per minute. Using the first-law idea, how many cars per minute must leave by the remaining exit road?

🧪 Nature of Science

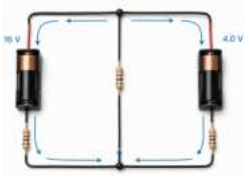
From the original — two conservation laws in disguise. Kirchhoff's laws are not new forces or new measurements — they are bookkeeping rules that follow from conservation principles already trusted across physics. The first (junction) law expresses conservation of electric charge: charge cannot pile up at a point, so whatever flows in must flow out. The second (loop) law expresses conservation of energy: a charge carried once around a closed loop returns to the same potential, so the potential supplied by the sources must equal the potential given up as voltage drops across the resistors.

Seeing one idea reappear as another is a habit of good science: a rule about traffic, a rule about altitude on a hike, and a rule about circuits are all the same conservation statement wearing different clothes. Recognizing that shared structure is what lets a single method solve problems that look unrelated.

Your turn: Suggest which conservation law each of Kirchhoff's two laws expresses, and name one other situation in physics where the same conservation law is used.

Lesson question, answered — In a branching circuit the currents and voltages obey two regularities, Kirchhoff's laws. At every junction the currents flowing in balance the currents flowing out (charge is conserved), and around

every closed loop the electromotive forces sum to the voltage drops RI (energy is conserved). Assigning a direction to each current and writing one junction equation and one loop equation for each independent loop gives enough simultaneous equations to find every current — and a negative answer simply reverses the assumed arrow.



The scene we opened with.

Exercise

1. A circuit consists of a battery and resistors as shown in Figure 3-2-9. Determine the direction and magnitude of the current flowing through each resistor.

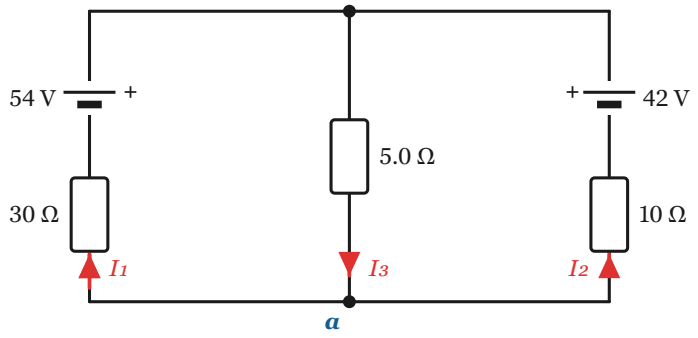


Figure 3-2-9 · Exercise 1: batteries 54 V and 42 V with resistors 30 Ω, 5.0 Ω, and 10 Ω in a two-loop network.

2. A circuit consists of a battery and resistors as shown in Figure 3-2-10. Determine the direction and magnitude of the current flowing through each resistor.

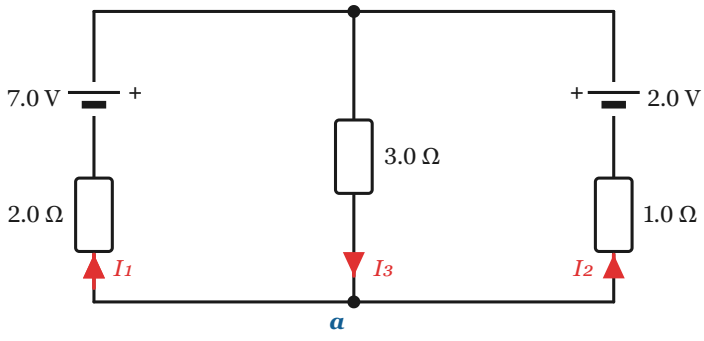


Figure 3-2-10 · Exercise 2: batteries 7.0 V and 2.0 V with resistors 2.0 Ω, 3.0 Ω, and 1.0 Ω in a two-loop network — the 7.0 V battery shares its branch with the 2.0 Ω resistor, the 2.0 V battery with the 1.0 Ω resistor, and the 3.0 Ω resistor is the shared middle branch.

3. A circuit consists of a battery and resistors R_1 to R_3 as shown in Figure 3-2-11. Determine the direction and magnitude of the current flowing through each resistor.

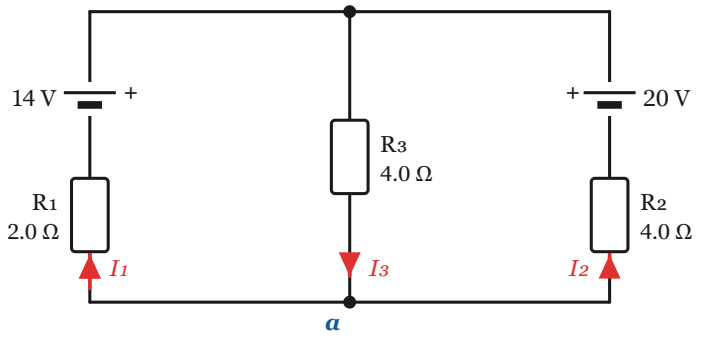


Figure 3-2-11 · Exercise 3: batteries 14 V and 20 V with R_1 = 2.0 Ω, R_2 = 4.0 Ω, R_3 = 4.0 Ω in a two-loop network — the 14 V battery shares its branch with R_1, the 20 V battery with R_2, and R_3 is the shared middle branch.

UNIT 4

Magnetism and Electromagnetic Induction

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4-4	Electromagnetic Induction	123
4-5	Self-Induction	130

4-1 Magnetic Field Created by Current

❓ **Lesson question:** How strong is the magnetic field that an electric current creates, and what sets its size?

You will learn to

1. Calculate the magnetic field strength H [A/m] at a distance r from a long straight current using $H = I/(2\pi r)$.
2. Find the magnetic field strength at the center of a circular current using $H = I/(2r)$, and multiply by the number of turns N for a coil.
3. Determine the magnetic field strength inside a solenoid from the turns per meter n using $H = nI$.

🔍 The phenomenon — predict before you continue

Lay a magnetic compass flat and let its needle settle along north–south. Now run a straight wire above the needle, pointing the same way, and switch on a current. The needle swings aside. Switch the current off and it returns; reverse the current and it swings the other way. The moving charge in the wire has surrounded itself with a magnetic field.

★ **Key idea**

Around a wire carrying a current I a magnetic field appears; its strength H is measured in amperes per meter [A/m]. How far does it reach, and which way does it lie? Your compasses will say.

Figure 4-1-1 · A straight current-carrying wire laid above a compass: while current I flows, the needle swings aside from north–south.

Predict

(a) If you double the current in the wire, does the deflection of the needle grow, shrink, or stay the same?

(b) If you move the compass farther from the wire, does the field it feels grow, shrink, or stay the same?

🔍 Explore — Oersted's compass circle

What you'll do: Read how a compass needle turns near a straight current as you change the current and the distance, uncovering that the field tracks I/r.

Time: 15 min

You will need:

- A stiff card with a vertical wire pushed through it
- Several small plotting compasses
- An ammeter to read the current I
- A ruler for the distance r and a protractor for the deflection θ
- A dry cell (with a rheostat to vary the current)

Card

Compasses

Ammeter

Dry cell

Steps

1. Stand the wire vertically through the card, scatter the small compasses around it, and let them settle along north–south with no current.
2. Switch on a known current I (read the ammeter) and, for one compass at distance r from the wire, record how far its needle turns from north, θ.
3. Raise I in steps at a fixed r, then hold I fixed and read compasses at increasing r; for each run work out the derived column I/r.
4. (Optional) Reverse the current and note that every needle swings the opposite way.

Run	Current I (A)	Distance r (m)	Deflection θ (°)	I/r (A/m)
1				
2				
3				

Analyze what you collected:

(a) Read the θ column against the I/r column: when I/r is larger, is the deflection larger too? What single combination of current and distance does the field seem to track?

(b) The straight-wire law is $H = I/(2\pi r)$. Predict from it: to get the same deflection, would doubling I and doubling r together cancel out? Do your runs agree?

✎ Write your explanation in two lines before you read the next page.

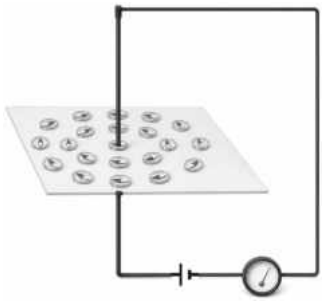


Figure 4-1-2 · A vertical wire through a card: small compasses line up on circles around the wire, mapping the field while the ammeter reads the current I.

★ **Working note**

The deflection angle θ from north is only a stand-in for the field — read it against the same north line every time, and keep iron and other magnets well away so nothing but the wire's field turns the needle.

From observation to model

A current builds a magnetic field around itself

Your compasses answered before any formula did. With the switch open every needle lay along north; close it and they all swung — so it is a current, and not any magnet, that fills the space around the wire. And they did not point at the wire or away from it: they settled along circles drawn round it, which is why this field is said to circle the current rather than to stream out of it. Reverse the current and every needle swings back the other way, so the sense of that circling follows the direction of the current — which is all the right-hand grip rule records. A deflection gives no

number, only a comparison, and the comparison held across every run: the needles turned further when you raised I, and less when you read them at a larger r, so what the field tracks is the single combination I/r. A current divided by a length is therefore what the model has to name, and that alone fixes its unit: the magnetic field strength H, in amperes per meter [A/m]. How the combination is dressed up — what number stands in front of it — is the one thing the compasses cannot settle, and it turns out to depend on the shape the wire is bent into.

Three standard geometries

Bend one wire three ways and a single source law shows three faces. Left **straight**, its field spreads evenly around a circle that widens as you step back, so the same current is shared out over an ever longer path and the strength thins away in proportion to how far off you stand — the 2π in the denominator is that circumference doing the sharing. Close the wire into a **ring** and every part of it now curves toward one point instead of running past it, so at that center only the radius is left to set the strength — there is no widening circle for the current to be shared out over, and the 2π loses the circumference it stood for. What the sharing picture cannot give is the number left in front: fixing that means adding up the contributions arriving at the center from every part of the loop, and that sum belongs to a later course. It settles at a bare $2r$ in place of $2\pi r$, which is to say a ring is π times stronger at its center than a straight wire is one radius away. Stack N such rings together and N equal contributions add there. Wind the wire on into a long cylinder and neighboring turns overlap so completely that, along the axis, the thinning stops altogether: what is left is the same everywhere inside, and the only thing that can raise it is packing more turns into each meter. Notice what drops out of that last result — the diameter of the cylinder never appears.

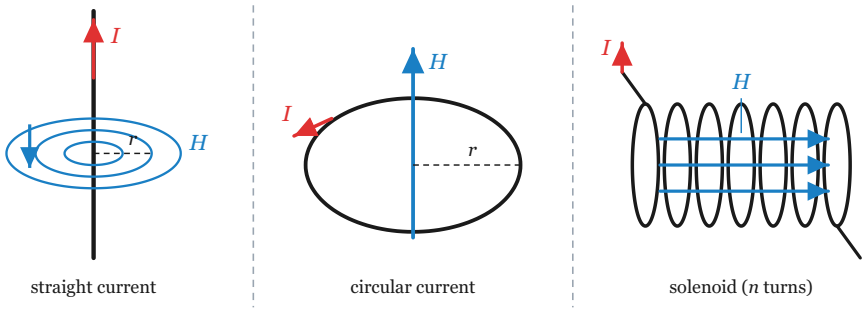


Figure 4-1-3 · Three geometries: the field circles a straight current, threads the center of a circular current, and runs uniform inside a solenoid.

Point

Magnetic field strength created by a straight current

Let the magnetic field strength be H [A/m], the current be I [A], and the distance from the current be r [m],

$$H = \frac{I}{2\pi r}$$

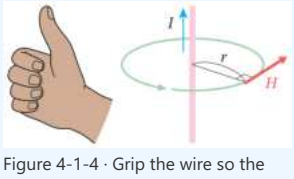


Figure 4-1-4 · Grip the wire so the thumb points along the current I; the fingers curl the way the field H circles it.

Magnetic field strength at the center created by a circular current

Let the magnetic field strength be H [A/m], the current be I [A], and the radius of the circular

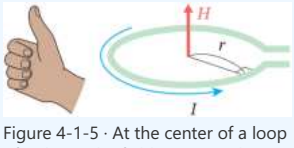


Figure 4-1-5 · At the center of a loop of radius r, the field H points along the axis while the current I runs round the ring.

★ **Key fact**

The scale of it: one ampere in a straight wire leaves about 0.16 A/m a meter away, and roughly 16 A/m a centimeter away. To reach the thousands of A/m an electromagnet needs, you do not raise the current much — you wind hundreds of turns into every meter.

★ **Term**

Solenoid — calling the inside field uniform is a model, and it is a fair one only well inside a cylinder much longer than it is wide. Approach either end and the overlap of turns runs out: the field there is weaker than the formula promises and splays outward instead of running straight.

★ **Note**

Two different counts, one easy slip: the ring result wants the plain number of turns, the cylinder result wants turns per meter, $n = N/L$. Feed a bare turn count into the solenoid formula and the answer comes out short by the factor L.

current be r [m],

$H = \dfrac{I}{2r}$

! Magnetic field strength inside a solenoid
A cylindrical coil of closely wound conducting wire is called a solenoid.
Let the magnetic field strength be H [A/m], the current be I [A], and the number of turns per 1 m be n [1/m],

$H = nI$

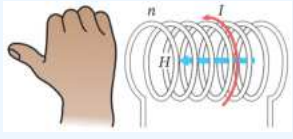


Figure 4-1-6 · Curl the fingers along the current I in the turns; the thumb points the way the field H runs along the axis.

Equation box

Straight current	$H = \dfrac{I}{2\pi r}$ (unit: A/m; r = distance from the wire)
Circular current (center)	$H = \dfrac{I}{2r} \Rightarrow N$-turn coil: $H = \dfrac{NI}{2r}$
Solenoid	$H = nI$ $(n = \dfrac{N}{L}$, turns per meter)

Worked Example

Situation	Determine each answer to 2 significant figures, assuming $\pi = 3.14$. (a) Determine the magnetic field strength in A/m at a point 1.0 m away from a straight current of 6.0 A. (b) When a current of 0.30 A flows through a 5-turn circular coil with a radius of 0.20 m, determine the magnetic field strength at the center of the circle in A/m. (c) When a current of 2.0 A flows through a solenoid with a length of 0.50 m and 300 turns, determine the magnetic field strength created inside it in A/m.
Given and required	(a) $I = 6.0$ A, $r = 1.0$ m; find H. (b) $I = 0.30$ A, $N = 5$, $r = 0.20$ m; find H at the center. (c) $I = 2.0$ A, length $L = 0.50$ m, $N = 300$ turns; find H inside.
Representation	See Figure 4-1-3: use the straight-wire panel for (a), the circular-loop panel for (b), and the solenoid panel for (c).
Strategy	Select the formula that matches each geometry — straight current $H = I/(2\pi r)$, circular current $H = I/(2r)$, solenoid $H = nI$ — and substitute the current, distance, or number of turns to determine the magnetic field strength. For a coil, multiply the single-turn result by N; for a solenoid, first find $n = N/L$.

Annotated solution	**(a)** Straight current, $H = I/(2\pi r)$: $H = 6.0 / (2\times3.14\times1.0) = 6.0/6.28 = 0.955\ldots \approx$ **0.96 A/m**. **(b)** In an N-turn circular coil, the field strength is N times that of a single-turn circular current, $H = NI/(2r)$: $H = 0.30 / (2\times0.20) \times 5 = 0.75 \times 5 = 3.75 \approx$ **3.8 A/m**. **(c)** Solenoid, $H = nI$ with $n = N/L = 300/0.50$: $H = (300/0.50) \times 2.0 = 600 \times 2.0 =$ **1.2×10^{3} A/m**.
Reasonableness check	(a) Dividing a few amperes by about 6 gives a field near 1 A/m — the right size for a wire a meter away. (b) Each turn adds 0.75 A/m, and five turns give a few A/m. (c) A tightly packed 600-turns-per-meter coil concentrates the field, so a value in the thousands of A/m is expected.
Explain it yourself	In one sentence, say why a circular coil produces a stronger field at its center than a single loop carrying the same current.

✍️ **Now you try:** Determine the magnetic field strength in A/m at a point 2.0 m away from a straight current of 8.0 A ($\pi = 3.14$, 2 significant figures).

🔆 Try

1. Determine the following to 2 significant figures, assuming $\pi = 3.14$. Determine the magnetic field strength in A/m at a point 0.50 m away from a straight current of 4.0 A.
2. When a current of 0.72 A flows through a 20-turn circular coil with a radius of 0.15 m, determine the magnetic field strength at the center of the circle in A/m.
3. When a current of 1.5×10^{-2} A flows through a solenoid with a length of 0.20 m and 200 turns, determine the magnetic field strength created inside it in A/m.

🌐 In a new context

Wind a solenoid, drop an iron core inside it, and switch on a current: it becomes an electromagnet whose field you can turn on and off and steer with the current direction. Because the field inside follows $H = nI$, packing in more turns per meter or raising the current makes it stronger — which is how relays snap contacts closed, how scrap-yard cranes grip steel, and how loudspeakers drive their cones.

➤ **Your turn:** A solenoid 0.40 m long is wound with 800 turns and carries 1.5 A. Determine the magnetic field strength inside it in A/m.

Figure 4-1-7 · A scrap-yard lifting electromagnet — a coil of many copper turns wound on an iron core, fed by an insulated power cable — grips a dense clump of steel scrap beneath its flat face and holds it against gravity, with only ferromagnetic material clinging on, densest at the face.

🧪 Nature of Science

From the original — is the Earth a giant electromagnet? The Earth behaves like a giant magnet, with the geographic North Pole acting as a south magnetic pole and the geographic South Pole as a north magnetic pole (geomagnetism). This field is thought to arise from the high-temperature liquid iron at the center of the Earth, which convects along with the Earth's rotation and drives a huge electric current inside.

The link Oersted found between current and field is one relationship expressed in several forms — one for each geometry — rather than many separate facts. Reading the Earth's field as evidence of an internal current shows how physicists extend a law tested on a benchtop wire to a system they can never open up, then test the extension against what they can measure at the surface.

Your turn: Suggest one observation at the Earth's surface that a scientist could use to test the idea that the planet's magnetism comes from an internal electric current.

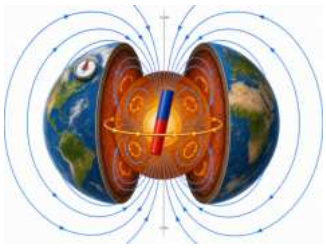


Figure 4-1-8 · The Earth as a giant electromagnet: a current in the convecting liquid-iron core builds a dipole field, and a surface compass lines up with it.

Lesson question, answered — A current always surrounds itself with a magnetic field whose strength H is measured in A/m. Its size is set by the current and the geometry: $H = I/(2\pi r)$ at distance r from a straight wire, $H = I/(2r)$ (times N for a coil) at the center of a circular current, and $H = nI$ inside a solenoid. Bigger current, tighter geometry, or more turns per meter all raise the field.

The scene we opened with.

📋 Exercise

1. Determine the following to 2 significant figures, assuming $\pi = 3.14$. Determine the magnetic field strength in A/m at a point 0.20 m away from a straight current of 3.14 A.
2. Determine the magnetic field strength in A/m at a point 0.20 m away from a straight current of 0.68 A.
3. When a current of 0.40 A flows through a circular conducting wire with a radius of 0.10 m, determine the magnetic field strength at the center of the circle in A/m.
4. When a current of 0.50 A flows through a 10-turn circular coil with a radius of 0.10 m, determine the magnetic field strength at the center of the circle in A/m.
5. When a current of 2.0 A flows through a solenoid with a length of 0.10 m and 500 turns, determine the magnetic field strength created inside it in A/m.
6. When a current flows through a solenoid with a length of 0.10 m and 250 turns, a magnetic field of 5.0×10^2 A/m was created inside it. Determine the current that flowed in A.

4-2 Force Exerted on a Current by a Magnetic Field

❓ **Lesson question:** How large is the push a magnetic field gives a current-carrying wire, and which way does that push point?

You will learn to

1. Calculate the force on a current perpendicular to a magnetic field using $F_B = \mu IHL$, and identify μ as the magnetic permeability [N/A^2] of the surrounding material.
2. Use the magnetic flux density $B = \mu H$ [T] to write the force as $F_B = IBL$, and use $I \sin\theta$ for a current that meets the field at an angle: $F_B = IBL \sin\theta$.
3. Apply Fleming's left-hand rule to find the direction of the force from the directions of the current and the magnetic field.

🔍 The phenomenon — predict before you continue

★ **Key idea**
A wire of length L carrying a current I across a field B [T] feels a force F_B [N]. How large, and which way?

Rest a fingertip lightly on the paper cone of a loudspeaker while music plays: you feel it flutter in and out. Inside, a coil of fine wire hangs in the narrow gap of a permanent magnet and is glued to the back of the cone. As the amplifier drives the music current back and forth through that coil, the magnetic field pushes the coil — and the cone with it — in and out, and the moving cone drives the air to make sound. Turn the volume up on the same note and the cone thumps harder. A current sitting in a magnetic field feels a push.

Figure 4-2-1 · A loudspeaker's voice coil hangs in a magnet's field: the music current through the coil makes the field push it — and the cone glued to it — in and out (force F_B), so a larger current gives a bigger push.

Predict

(a) If you turn the volume up so the current in the coil is larger, does the push on the cone grow, shrink, or stay the same?

(b) If the coil's wire instead ran along the magnetic field rather than across it, would the push grow, shrink, or disappear?

🔍 Explore — Weighing the push on a current

What you'll do: Measure the sideways force on a current-carrying rod as a weight change on a balance, and see how that force grows with the current.

Time: 15 min

You will need:

- Magnet with N and S poles
- Sensitive balance
- DC supply
- Ammeter

Magnet poles | Sensitive balance | DC supply | Ammeter

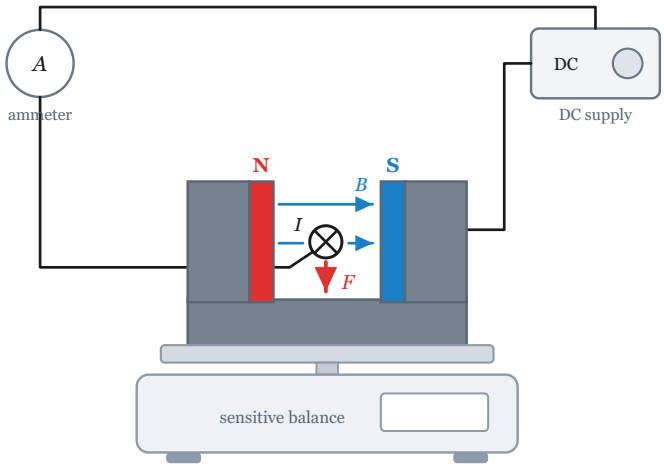


Figure 4-2-2 · A conducting rod of length L carrying current I rests between the poles of a magnet on a sensitive balance, which reads the force. A variable DC supply and an ammeter set and measure I. The rod runs into the page, at right angles to B, so the force on it is vertical and the balance reads it.

Steps

1. Clamp the magnet with its N and S poles facing across a gap, and rest a straight conducting rod of length L across the gap at right angles to the field.
2. Stand the magnet assembly on the pan of the sensitive balance and zero the reading with no current flowing.
3. Close the circuit through the DC supply and ammeter, set a current I, read the force F_B from the change in the balance, then compute F_B/I.
4. Raise I in equal steps and repeat, keeping the rod perpendicular to the field.

> ★ **Working note**
> Zero the balance with the current switched off, so each reading is the change caused by the current alone; a mass change Δm means a force $F_B = \Delta m \times g$.

Run	Current I (A)	Force F_B (mN)	F_B/I (mN/A)
1			
2			
3			

Analyze what you collected:

(a) Look down the F_B/I column: how does its value behave as the current changes, and what does a constant F_B/I tell you about how the force depends on I?

(b) For this gap the constant F_B/I equals BL. Using the length L you set, predict the force at a current you did not test, and say what a stronger magnet would do to the value.

✎ Write your explanation in two lines before you read the next page.

From observation to model

A magnetic field pushes on a current

The Explore run settles the shape of the answer. The push grew step for step with the current, and the ratio you tabulated held steady as the current changed — so the force is proportional to *I*. Two more factors have to sit in the same product for the same reason: a longer stretch of wire inside the gap holds proportionally more moving charge to be pushed, and a stronger field pushes each carrier proportionally harder. That fixes the force as a product of current, field strength and immersed length, with one constant left over to carry the effect of whatever fills the gap — the permeability μ, which is why an iron-filled gap gives a far larger push than an air gap at the same *H*. Swing the wire away from the perpendicular and nothing about the field changes, only the geometry: the part of the current running *along* the field lines is not pushed at all, so only its across-the-field component survives, shrinking as sin θ until a wire laid parallel to the field feels no push whatever.

Magnetic flux density and the tesla

Carrying μ and *H* around separately is a nuisance, because the same current in the same gap gives a different push depending on what is packed into it. The two always appear together as a product, so they may as well be bundled into a single quantity, and that bundle is what gets its own name and its own unit. Making the swap shortens both force relations to a form with no μ left in sight, and the practical gain is large: the bundled quantity is exactly what a force measurement returns. Weigh the push on a known current in a known length, divide, and the gap has reported its own value — whereas *H* can be inferred only once you know what material is in the way. The tesla is defined by that very measurement, one newton per ampere-meter.

The direction of the force

Nothing in the size of the force says which way the wire will move, and that has to be settled separately. What the geometry actually asserts is that the three directions — current, field, and push — are mutually perpendicular, so the push always leaves the plane the other two define. Fleming's rule is simply a way of holding that fact in one hand: three fingers of the left hand spread at right angles reproduce the arrangement, and reading them off answers the question without redrawing anything. Two consequences follow at once. Reverse either the current or the poles and the push reverses; reverse both and it does not. And because the push so often points straight out of the page or straight into it, this lesson leans on the two depth symbols of the Point far more than on any arrow.

★ **Key fact**

The scale of it: one ampere through one meter of wire lying across a one-tesla field is pushed with one newton — about the weight of a small apple. Bench magnets manage only a few hundredths of a tesla, which is why the pushes you weigh in the laboratory come out in millinewtons.

★ **Term**

Magnetic permeability — treating μ as one fixed number is what keeps the model simple, and it is safe for air and for ordinary non-magnetic surroundings. Iron and the other ferromagnets break it: their permeability itself depends on the field already applied to them, and it collapses once the material saturates, so the straight-line link between *H* and *B* stops holding.

★ **Note**

It is the *left* hand here. A right-hand version exists but answers a different question — the direction of an induced current, met two lessons on — and reaching for the wrong hand reverses every answer. Note too that the rule fixes a direction only; the sin θ factor must still be carried in the magnitude.

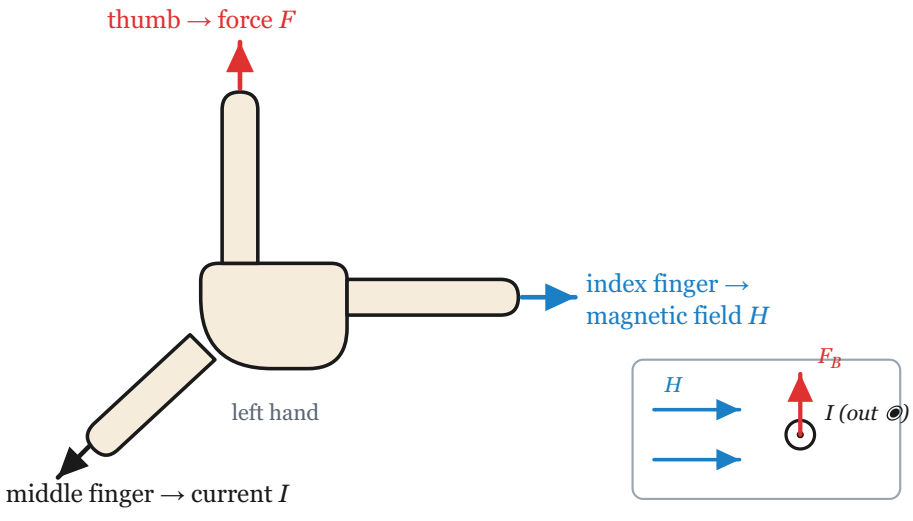
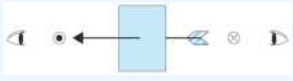


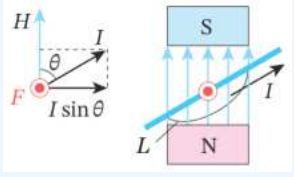
Figure 4-2-3 · Fleming's left-hand rule: with the thumb, index finger, and middle finger held at right angles, the middle finger points along the current I, the index finger along the magnetic field H, and the thumb gives the force F_B — as in the inset rod carrying current out of the page across a rightward field.

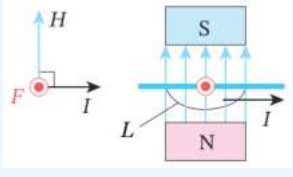
Point

- Symbols representing the direction perpendicular to the page
  The direction out of the page is represented by (◉), and the direction into the page is represented by (⊗).



- Let the magnitude of the force exerted on the current by the magnetic field be F_B [N], the current be I [A], the magnetic field strength be H [A/m], and the length of the conducting wire in the magnetic field be L [m],





  • When the current and the magnetic field intersect perpendicularly
  F_B = ***μIHl***
  μ is called the **magnetic permeability** [N/A^2], and is determined by the type of surrounding material.
  • When the current and the magnetic field are not orthogonal
  As the current value, the component perpendicular to the magnetic field, $I \sin\theta$, is used.

- The quantity representing the state of the magnetic field including the influence of surrounding materials is called **magnetic flux density**, defined as $B = \mu H$.
  The unit of magnetic flux density used is the **tesla** (symbol **T**).

- Expressing the magnitude of the force F_B [N] exerted on the current by the magnetic field using the magnetic flux density B [T],
  When the current and the magnetic field intersect perpendicularly,
  F_B = ***IBL***
  When the current and the magnetic field intersect obliquely,
  F_B = ***IBL sin θ***

- Fleming's left-hand rule
  Direction of the middle finger → Direction of **current**
  Direction of the index finger → Direction of **magnetic field**
  Direction of the thumb → Direction of **force**

Equation box	
Force (field strength H)	$F_B = \mu IHL$ (perpendicular) $\Longrightarrow$ $F_B = \mu IHL \sin\theta$ (at angle θ)
Magnetic flux density	$B = \mu H$ (unit: tesla, T)
Force (flux density B)	$F_B = IBL$ (perpendicular) $\Longrightarrow$ $F_B = IBL \sin\theta$ (at angle θ)

🛠 Worked Example

Situation	A conducting rod PQ with a length of 0.10 m is placed in a uniform magnetic field of 50 A/m directed to the right, at an angle of 30° to the magnetic field, and a current of 1.2 A flows in the direction from P to Q. Determine the direction and magnitude F_B [N] of the force exerted on the conducting rod PQ by the magnetic field, to 2 significant figures. Assume the magnetic permeability is 1.26×10^{-6} N/A².
Given and required	I = 1.2 A, H = 50 A/m, L = 0.10 m, θ = 30°, $\mu = 1.26\times10^{-6}$ N/A². Find the direction of F_B and its magnitude to 2 significant figures.
Representation	See Figure 4-2-5: the rod PQ carries current I at 30° to a horizontal field H; the component of the current across the field is I sin 30°.
Strategy	Find the direction of the force with Fleming's left-hand rule (middle finger → current, index finger → field, thumb → force). Then find the magnitude: the current meets the field at an angle, so use the perpendicular component I sin θ in $F_B = \mu IHL \sin\theta$.
Annotated solution	**Direction.** By Fleming's left-hand rule, the force is perpendicular to the page, directed out of the page (◉). **Magnitude.** The current meets the field at 30°, so use $F_B = \mu IHL \sin\theta$: $F_B = (1.26\times10^{-6}) \times 1.2 \times \sin 30° \times 50 \times 0.10$ $= (1.26\times10^{-6}) \times 1.2 \times 0.50 \times 50 \times 0.10 = 3.78\times10^{-6} \approx$ **3.8×10^{-6} N**.
Reasonableness check	The magnetic permeability of about 10^{-6} makes the force very small, and the factors of order 1 (current) and 0.1 (length) keep it near a few $\times10^{-6}$ N — the right size for a bench wire in a weak field. Because the angle is only 30°, the $\sin\theta$ = 0.50 factor halves what a perpendicular current would feel.

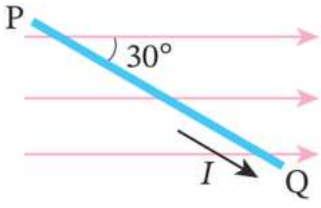


Figure 4-2-4 · Worked Example: the rod PQ lies at 30° to a magnetic field directed to the right, with the current I running from P to Q.

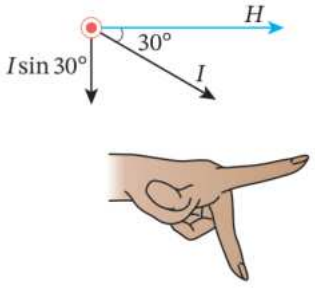


Figure 4-2-5 · Worked Example: the current I crosses the field H at 30°, so only the perpendicular component I sin 30° drives the force; the hand shows Fleming's left-hand rule.

🧐 **Explain it yourself**	In one sentence, say why the force would be largest if the rod were turned to sit perpendicular to the field instead of at 30°.

✍️ **Now you try:** A conducting rod of length 0.10 m is placed in a uniform magnetic field of 40 A/m at an angle of 30° to the field, and a current of 2.0 A flows through it. Determine the magnitude of the force to 2 significant figures ($\mu = 1.26\times10^{-6}$ N/A^2).

🔆 Try

1. Determine the following to 2 significant figures. A conducting wire is stretched perpendicularly to a magnetic field in a uniform magnetic field of 20 A/m, and a current of 4.0 A flows through it. Determine the magnitude of the force exerted on a 0.10 m portion of the conducting wire in N. Assume the magnetic permeability is 1.26×10^{-6} N/A^2.
2. As shown in the figure, a conducting rod PQ with a length of 0.10 m is placed in a uniform magnetic field with a magnetic flux density of 2.0×10^{-2} T directed to the left, at an angle of 30° to the magnetic field, and a current of 1.2 A flows in the direction from P to Q. Determine the direction and magnitude F_B [N] of the force exerted on the conducting rod PQ by the magnetic field.

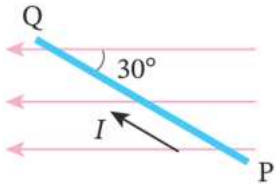


Figure 4-2-6 · Try: the straight conductor PQ carries a current I across a uniform field whose lines run to the left, meeting them at 30°.

🌐 In a new context

The motors in an electric fan, an electric vehicle, and countless other machines turn using this very force. A current is passed through a coil sitting between the poles of a magnet; the magnetic field pushes on the two sides of the coil in opposite directions, and the pair of forces spins the coil. As the coil rotates past the vertical, the direction of the current through it is switched, so the push always acts to keep it turning the same way. Steady rotation from a magnetic field is just the force $F_B = IBL$ applied around a loop, over and over.

➤ **Your turn:** One side of a motor coil, 0.050 m long, lies perpendicular to a field of magnetic flux density 0.20 T and carries 3.0 A. Determine the force on that side of the coil in N.

🧪 Nature of Science

From the original — Defining a quantity by the force it produces. The magnetic flux density B is not read straight off an instrument; it is defined through the relation $F_B = IBL$. Physicists fix a known current I in a known length L placed perpendicular to the field, measure the force F_B, and solve $B = F_B/(IL)$. The tesla is then the field in which a 1 A current in a 1 m wire feels a force of 1 N.

This is an operational definition: a quantity is pinned down by the measurement that produces it, not by a picture of what the field "is." Writing $B = \mu H$ keeps the connection to the surrounding material explicit, while $F_B = IBL$ keeps B tied to something you can weigh on a balance.

Ampère had this force measured within weeks of hearing of Oersted's compass in 1820. Fleming's rule came much later and is only a mnemonic — a way to remember the geometry, not a law of nature; the physics is the vector relation the rule stands in for.

Your turn: Suggest one measurement you could make on a current-carrying wire between magnet poles to determine the magnetic flux density B of the gap.

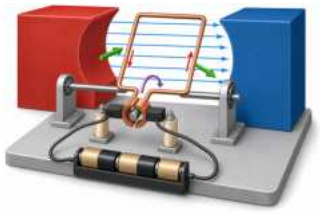


Figure 4-2-7 · A simple DC motor: current I in a coil between the N and S poles feels opposite forces $F_B = IBL$ on its two sides, which turn the coil; a commutator reverses the current each half-turn so the rotation keeps going the same way.

Lesson question, answered — A current in a magnetic field feels a force because the field pushes on the moving charge, and the wire moves under it. When the current and field are perpendicular the force is $F_B = \mu IHL = IBL$, where $B = \mu H$ is the magnetic flux density in tesla; when they meet at an angle θ, only the component $I \sin\theta$ counts, so $F_B = IBL \sin\theta$. Its direction is set by Fleming's left-hand rule. Bigger current, stronger field, or longer wire all increase the force.

The scene we opened with.

Exercise

1. Determine the following to 2 significant figures. A conducting wire is stretched perpendicularly to a magnetic field in a uniform magnetic field of 20 A/m, and a current of 1.0 A flows through it. Determine the magnitude of the force exerted on a 0.20 m portion of the conducting wire in N. Assume the magnetic permeability is 1.26×10^{-6} N/A^2.
2. A conducting wire is stretched perpendicularly to a magnetic field in a uniform magnetic field of 25 A/m, and a current of 2.0 A flows through it. Determine the magnitude of the force exerted on a 0.10 m portion of the conducting wire in N. Assume the magnetic permeability is 1.26×10^{-6} N/A^2.
3. A conducting wire is stretched perpendicularly to a magnetic field in a uniform magnetic field with a magnetic flux density of 0.50 T, and a current of 4.0 A flows through it. Determine the magnitude of the force exerted on a 0.20 m portion of the conducting wire in N.
4. A conducting wire is stretched perpendicularly to a magnetic field in a uniform magnetic field with a magnetic flux density of 2.0×10^{-3} T, and a current of 5.0 A flows through it. Determine the magnitude of the force exerted on a 2.0 m portion of the conducting wire in N.
5. As shown in the figure, a conducting rod PQ with a length of 0.20 m is placed in a uniform magnetic field of 50 A/m directed to the right, at an angle of 30° to the magnetic field, and a current of 1.0 A is passed in the direction from P to Q. Determine the direction and magnitude F_B [N] of the force exerted on the conducting rod PQ by the magnetic field. Assume the magnetic permeability is 1.26×10^{-6} N/A^2.

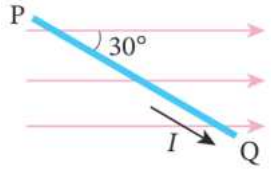


Figure 4-2-8 · Exercise 5: the conducting rod PQ lies at 30° to a uniform magnetic field directed to the right, with the current I flowing from P to Q.

4-3 Motion of Charged Particles in a Magnetic Field

❓ **Lesson question:** What force does a magnetic field exert on a moving charge, and what path does that charge then follow?

You will learn to

1. Calculate the magnitude of the Lorentz force on a charge moving perpendicular to a magnetic field using $f = qvB$.
2. Use Fleming's left-hand rule — taking the equivalent current along the motion of a positive charge, and opposite to the motion of a negative charge — to find the direction of the Lorentz force.
3. Set the Lorentz force equal to the centripetal force, $mv^2/r = qvB$, to find the radius $r = mv/(qB)$ and the period $T = 2\pi m/(qB)$ of the circular path.

🔍 The phenomenon — predict before you continue

Send a thin electron beam across the gap between two magnet poles. With the magnets away, the beam races straight to a glowing spot on the far screen. Switch the field on and the spot swings aside; strengthen the field and the beam bends into a tighter arc, until it curls into a full circle that loops back on itself. Faster electrons trace wider circles, slower ones tighter — yet nothing touches the beam but the field.

★ **Key idea**

Lorentz force f [N] — what a field B does to one charge q moving at speed v. How big, which way, and what path does it force?

Figure 4-3-1 · With the field off the electron beam runs straight to the screen; switch on a field into the page and it curls into a circular path.

Predict

(a) If you strengthen the magnetic field, does the radius of the electron's circular path grow, shrink, or stay the same?

(b) If the electrons enter faster, does the time to complete one full loop grow, shrink, or stay the same?

🔍 Explore — Bending an electron beam into a circle

What you'll do: Steer a fine electron beam with a magnetic field and measure how the radius of its circular path depends on the beam's speed and on the field strength.

Time: 20 min

You will need:

- Fine-beam (e/m) tube
- Helmholtz coils
- DC supply
- Ruler
- A voltmeter for the accelerating voltage and an ammeter for the coil current

Fine-beam tube

Helmholtz coils

DC supply

Ruler

Steps

1. Set the accelerating voltage so the electrons leave the gun at a known speed v, and switch on the Helmholtz coils to fill the tube with a uniform field B perpendicular to the beam.
2. Adjust the coil current until the glowing beam closes into a full circle; read its diameter $2r$ on the ruler and halve it to get the radius r.
3. Hold v fixed and raise B, re-reading r; then hold B fixed and raise v, re-reading r. For each run record v, B, and r, and compute $r{\cdot}B/v$.
4. Finally, compare the ratio r/v between your fixed-B runs: if it does not change, the time for one loop, $2\pi r/v$, does not depend on the speed either.

Run	Speed v (m/s)	Field B (T)	Radius r (m)	$r{\cdot}B/v$
1				
2				
3				

Analyze what you collected:

(a) As you change the speed v and the field B from run to run, what happens to the value of $r{\cdot}B/v$ in the last column?

(b) The beam curls into a closed circle yet never speeds up — what does that say about the direction of the field's force, and how does a constant $r{\cdot}B/v$ point to $r = mv/(qB)$?

✎ Write your explanation in two lines before you read the next page.

Figure 4-3-2 · A fine-beam (e/m) tube: an electron gun fires a beam that a coil field bends into a glowing circle, whose diameter $2r$ is read off a ruler across the tube.

★ **Working note**

Get the speed from the accelerating voltage V_a using $\tfrac{1}{2}mv^2 = eV_a$, and the field from the coil current using the B-per-ampere constant printed on the Helmholtz pair. Read the beam's diameter $2r$ straight across the tube against the ruler and halve it to get r; sight the far edge of the glowing ring square-on to avoid a parallax error.

From observation to model

A magnetic field pushes a moving charge sideways

Nothing new has to be assumed here: a current is only charge in transit, so the push the last lesson measured on a whole wire must be the sum of pushes on the individual carriers inside it. Divide that wire force by the number of carriers the immersed length holds, and one particle's share is left. It has to grow with how much charge the particle carries and with how fast that charge cuts across the field lines,

and the wire result fixes the constant of proportionality as the flux density itself — which is the **Lorentz force**,

$$f = qvB$$

— for a particle crossing the field at right angles. The direction needs no new rule either: swap the particle for the short length of current it stands for and the same left hand answers. One trap is worth naming. Conventional current runs the way positive charge travels, so for an electron the stand-in current points backwards along the motion, and the push comes out on the opposite side from a positive particle following the same path.

A sideways push turns motion into a circle

Now ask what a push of unchanging size that stays permanently sideways can do to a moving particle. It can never add to the speed or take from it, because it never has a component along the velocity — it does no work, which is why the beam in the tube glowed no brighter as it curved. All it can do is steer. A push of constant magnitude that keeps swinging round to stay square to the motion is exactly the condition for turning at a constant rate, so the path closes on itself. Setting that push equal to the centripetal requirement,

$$\frac{mv^2}{r} = qvB$$

— one factor of the speed cancels straight away and leaves

$$r = \frac{mv}{qB}$$

— the speed decides how wide the circle is, not whether there is one. Carry that radius into the time for one lap and the speed cancels a second time, which is the surprise the Explore run pointed at — double the speed and the particle merely runs a circle of double the size, returning in precisely the same time. So mass, charge and field alone fix the period.

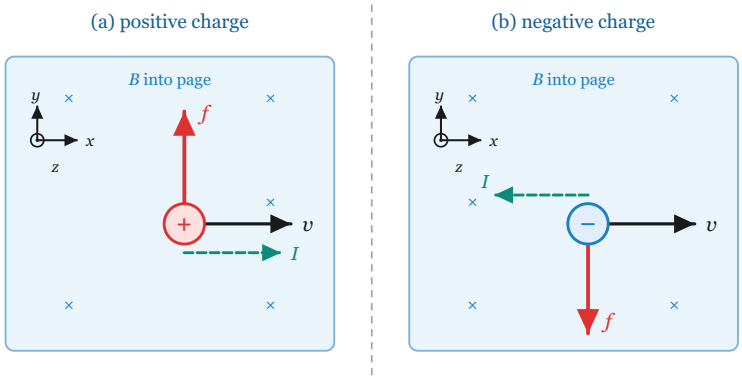
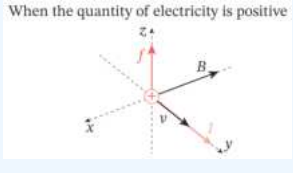


Figure 4-3-3 · Lorentz force in a field into the page: for a positive charge the equivalent current *I* runs along *v* and *f* points up; for a negative charge *I* runs opposite *v* and *f* points down.

Point

! The force exerted on a charged particle (a particle with charge) moving in a magnetic field is called the **Lorentz force**.
Let the magnitude of the Lorentz force be f[N] when a charged particle with a charge q[C] is moving at a speed v[m/s] in a direction perpendicular to the magnetic field in a magnetic field with a magnetic flux density B[T],
$f =$ **qvB**



! For the direction of the Lorentz force, assume a corresponding current I and use Fleming's left-hand rule.

★ **Term**

Lorentz force — the expression $f = qvB$ is the square-on case and no other. A particle drifting *along* the field lines feels nothing at all, and one entering obliquely is pushed only by the across-the-field part of its velocity, so it keeps its along-the-field drift and winds forward on a helix instead of closing a circle.

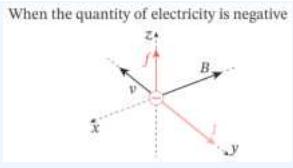
★ **Key fact**

The scale of it: an electron accelerated through 1000 V is traveling at about 1.9×10^{7} m/s, and a field of only 1.0×10^{-3} T bends it onto a circle roughly 0.11 m across the radius — a loop you can see from across the bench. A proton at the same speed, 1800 times heavier, would need 1800 times the field to be curled as tightly.

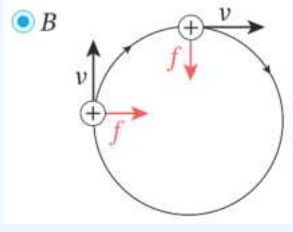
★ **Note**

Substitute the size of the charge, never its sign: the sign belongs to the direction argument, not to the magnitude of the force. And the circle lasts only as long as the particle stays inside the field — the instant it crosses the boundary no force acts on it and it leaves along the tangent, in a straight line.

When using the left-hand rule, pay attention to the sign of the charge and the direction of the current.



! When a charged particle enters a uniform magnetic field perpendicularly, the Lorentz force always acts in a direction **perpendicular** to the direction of motion of the charged particle.
⇒ The charged particle undergoes **uniform circular motion** in a plane perpendicular to the magnetic field.



! Equations for uniform circular motion

Equation of motion : $\boldsymbol{mv^2/r} = F$

Period T : $T =$ $\boldsymbol{\frac{2\pi r}{v}}$

Equation box

Lorentz force	$f = qvB$ (unit: N; v perpendicular to B)
Direction	Fleming's left-hand rule, current I along v (positive charge) or opposite to v (negative charge)
Circular motion	$\frac{mv^2}{r} = qvB \implies r = \frac{mv}{qB}$
Period	$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$

🛠 Worked Example

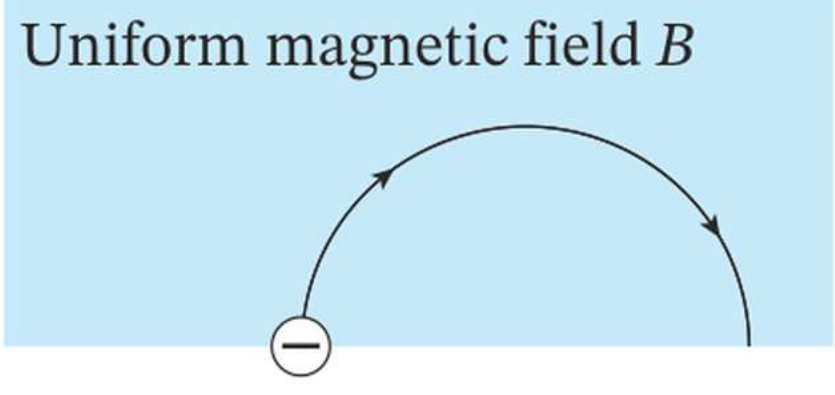


Figure 4-3-4 · Worked Example: an electron enters a uniform field B perpendicular to the page and describes a semicircle inside it.

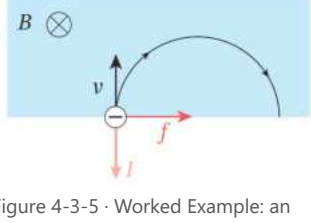


Figure 4-3-5 · Worked Example: an electron enters a field B directed into the page at velocity v; its charge is negative, so the equivalent current I runs opposite to v and the force f points to the center of the circle.

Situation	An electron of mass m [kg] and charge $-e$ [C] enters a uniform magnetic field of flux density B [T] that is perpendicular to the page. It enters perpendicularly at speed v [m/s] and describes a semicircle. (a) Determine the direction of the magnetic field. (b) Determine the magnitude of the force in N that the electron experiences from the magnetic field. (c) Determine the radius in m of the semicircle described by the electron. (d) Determine the time in s it takes for the electron to travel the semicircle.
Given and required	Given: mass m, charge $-e$, field B perpendicular to the page, entry speed v perpendicular to B. Required: (a) the field direction, (b) the force magnitude f, (c) the radius r, (d) the semicircle time t.
Representation	See Figure 4-3-5: the electron enters the field and curves along a semicircle; the Lorentz force f points toward the center of that circle at every instant.
Strategy	Follow the standard steps for a charged particle in a magnetic field: find the Lorentz force $f = qvB$ on the particle, set it equal to the centripetal force in the equation of motion $mv^2/r = qvB$, and from that solve for the orbital radius r and the period T. Fix the field direction with Fleming's left-hand rule, remembering that the electron's charge is negative so its equivalent current runs opposite to v.
Annotated solution	**(a)** By Fleming's left-hand rule, the Lorentz force on the electron points toward the center of its circular path. The electron's charge is negative, so the equivalent current I runs opposite to v. Applying the rule with that current gives the field perpendicular to the page, directed **into the page**. **(b)** Taking the magnitude of the charge as e (ignoring the sign): $f = evB \implies \boldsymbol{f = evB}$ **[N]**. **(c)** The Lorentz force is the centripetal force, so $mv^2/r = evB$. Solving for r: $r = mv^2/(evB) = \boldsymbol{mv/(eB)}$ **[m]**. **(d)** The semicircle takes half a period, $t = T/2 = \pi r/v$. Substituting $r = mv/(eB)$: $t = \pi\cdot(mv/(eB))/v = \boldsymbol{\pi m/(eB)}$ **[s]**.

Reasonableness check	(b) The force grows with charge, speed, and field, as $f = evB$ should. (c) A stronger field B gives a smaller radius and a faster electron a larger one, matching the curved-beam observation. (d) The semicircle time $\pi m/(eB)$ contains no v, so faster and slower electrons take the same time to complete the loop — exactly what the constant r/v found in Explore implies.
Explain it yourself	In one sentence, explain why the Lorentz force changes the direction of the electron's velocity but never its speed.

✍️ **Now you try:** A charged particle of charge q and mass m enters a uniform field of flux density B at speed v, perpendicular to the field. Determine (i) the magnitude of the Lorentz force it feels and (ii) the period of its circular motion.

🔆 Try

1. As shown in Figure 4-3-6, when a positively charged particle with a mass of m [kg] and a charge of q [C] enters a uniform magnetic field perpendicular to the page with a magnetic flux density B [T] perpendicularly at a speed v [m/s], it describes a semicircle. Determine the direction of the magnetic field.
2. Determine the time in s it takes for the charged particle to travel the semicircle?
3. In order for the charged particle not to cross the part (length l) in the figure and go to the right side, determine the maximum value of the incident speed v in m/s.

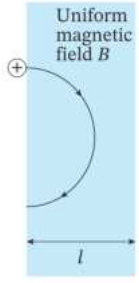


Figure 4-3-6 · Try: a positive charge enters a uniform field B that fills a strip of width l and turns through a semicircle inside it.

🌐 In a new context

Because the radius $r = mv/(qB)$ depends on a particle's mass and charge, a magnetic field acts as a sorter of charged particles. Send ions of the same charge and speed into a uniform field and each mass curves onto its own radius — heavier ions swing wide, lighter ones curl tight — so where each beam lands reveals its mass. This is the working principle of the mass spectrometer, and the same physics steers particle beams around the ring of a circular accelerator.

➤ **Your turn:** Two ions carry the same charge q and enter the same field B at the same speed v, but one has twice the mass of the other. Determine the ratio of their circular radii.

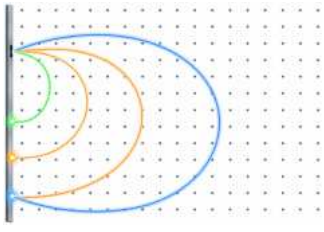


Figure 4-3-7 · A mass spectrometer: ions of the same charge but different mass follow circular arcs of different radii, the heaviest curving least.

🧪 Nature of Science

From the original — the aurora and the Lorentz force. The beautiful "aurora" seen in the polar regions is tied to the Lorentz force. High-energy charged particles streaming from the Sun (the solar wind) are captured by the Earth's magnetic field and guided toward the North and South Poles, bent into spiral paths by the Lorentz force. Where they collide with oxygen and nitrogen atoms in the atmosphere, the atoms emit light, and that glow forms the aurora.

This shows a law tested on a benchtop beam explaining a sky-wide display: the same $f = qvB$ that curls an electron in a tube also curves solar-wind particles onto the planet's field lines. Physicists trust the extension because the aurora appears exactly where the model predicts — over the magnetic poles, where the field guides charged particles down into the air.

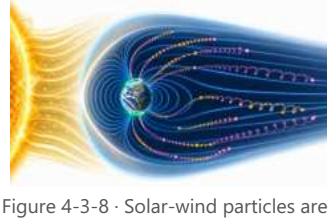


Figure 4-3-8 · Solar-wind particles are captured by Earth's magnetic field and spiral along the field lines toward the poles, where they light the atmosphere as an aurora.

Your turn: Suggest one observation about where or when auroras appear that a scientist could use to test the idea that the Lorentz force guides solar-wind particles toward the poles.

Lesson question, answered — A magnetic field exerts the Lorentz force $f = qvB$ on a charge moving across it, directed by Fleming's left-hand rule (current along v for a positive charge, opposite for a negative one). Because that force stays perpendicular to the velocity, it turns the charge without changing its speed, so a particle entering a uniform field perpendicularly moves in a circle of radius $r = mv/(qB)$ with period $T = 2\pi m/(qB)$.

The scene we opened with.

Exercise

1. When a positively charged particle with a mass of m [kg] and a charge of q [C] enters a uniform magnetic field perpendicular to the page with a magnetic flux density B [T] perpendicularly at a speed v [m/s], it describes a semicircle. (i) Determine the direction of the magnetic field. (ii) Determine the time in s it takes for the particle to travel the semicircle.
2. When an electron with a mass of m [kg] and a charge of $-e$ [C] enters a uniform magnetic field perpendicular to the page with a magnetic flux density B [T] perpendicularly, it describes a semicircle with a radius of a [m]. (i) Determine the direction of the magnetic field. (ii) Determine the incident speed of the electron in m/s.
3. As shown in Figure 4-3-11, when a positively charged particle with a mass of m [kg] and a charge of q [C] enters the magnetic field from a position at a distance of r [m] from A at a speed of v [m/s] perpendicularly to AB and perpendicularly to the magnetic field, the charged particle flies out of the magnetic field perpendicularly to AD from between A and D, at a distance of r [m] from A. (i) Determine the direction of the magnetic field. (ii) Determine the magnitude of the magnetic flux density in T? (iii) Determine the time in s it takes from when the charged particle enters the magnetic field until it flies out of the magnetic field.

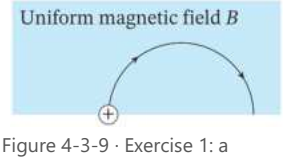


Figure 4-3-9 · Exercise 1: a positive charge enters the uniform field B from below and describes a semicircle inside it.

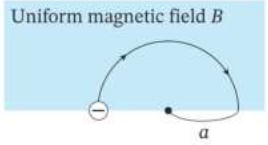


Figure 4-3-10 · Exercise 2: the electron enters the uniform field B and turns through a semicircle of radius a.

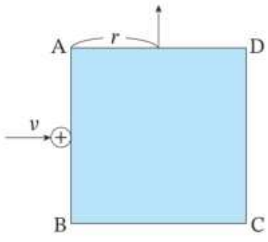


Figure 4-3-11 · Exercise 3: the particle crosses AB at a distance r from A, moving perpendicular to AB, and leaves across AD at the same distance r from A.

4-4 Electromagnetic Induction

❓ **Lesson question:** Why does a current flow in a coil simply by moving a magnet closer to or farther from it, with no battery connected?

You will learn to

1. Define magnetic flux as $\Phi_B = BA$, the flux density B times an area A held perpendicular to the field, measured in webers (Wb).
2. Apply Faraday's law of induction, $\varepsilon = -N\,\Delta\Phi_B/\Delta t$, to calculate the magnitude of the induced electromotive force in a coil of N turns.
3. Use Lenz's law to determine the direction of the induced current, so that its magnetic flux opposes the change in the external flux.

🔍 The phenomenon — predict before you continue

A coil is joined into a closed loop with a sensitive galvanometer, and no battery is in the circuit. When a bar magnet is held still inside or beside the coil, the galvanometer reads zero. Yet the moment the magnet is pushed in or pulled out, the needle swings, and it swings the other way when the motion reverses. Hold the magnet still again and the reading falls back to zero.

★ **Key idea**

Magnetic flux Φ_B [Wb] — flux density B times the area A it threads. What must Φ_B do before a circuit shows an emf, and what sets its size?

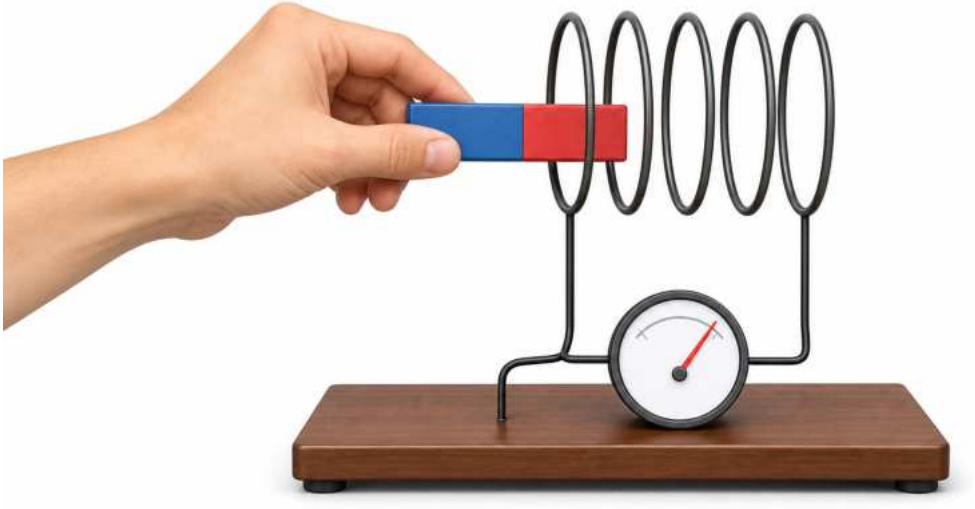


Figure 4-4-1 · A coil joined only to a galvanometer, with no battery; moving the bar magnet along the coil axis swings the needle while the field through the coil changes.

Predict

(a) Does the current appear because the magnet is near the coil, or because the magnetic field through the coil is *changing*?

(b) If you double the speed of the same magnet, what happens to the size of the deflection, and why?

🔍 Explore — Speed of a magnet vs the galvanometer swing

What you'll do: Move the same bar magnet through a coil at different speeds and see how the rate of flux change controls the size and direction of the induced emf.

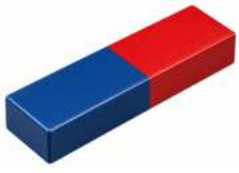
Time: 15 min

You will need:

- A coil of many turns
- A center-zero galvanometer
- A bar magnet
- A ruler, a stopwatch, and connecting leads

Coil

Galvanometer


Bar magnet

Ruler

Steps

1. Connect the coil to the galvanometer only, with no battery, and set the pointer to zero at the center.
2. Mark a fixed travel distance on the bench with the ruler; with the magnet held still outside, confirm the pointer reads zero.
3. Push the magnet through that same distance into the coil — once slowly, once quickly — recording the time Δt and the peak deflection (size and side); then hold it still inside and watch the reading fall back to zero.
4. Pull the magnet out over the same distance, note the swing now goes the opposite way, and work out $1/\Delta t$ for each run.

Run	Action (in / still / out)	Time to move Δt (s)	Peak deflection (div, L/R)	Rate $1/\Delta t$ (s^{-1})
1				
2				
3				

Analyze what you collected:

(a) As the rate $1/\Delta t$ grows (the same magnet moved faster over the same distance), what happens to the peak deflection? How does the "still" run compare?

(b) Since the same magnet over the same distance gives the same $\Delta\Phi_B$, what does the pattern reveal about $\varepsilon = -N\,\Delta\Phi_B/\Delta t$ — and why do pushing in and pulling out swing the needle opposite ways?

✎ Write your explanation in two lines before you read the next page.

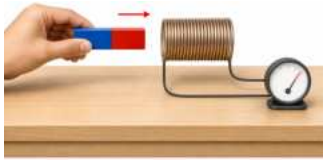


Figure 4-4-2 · Bench setup: a many-turn coil is wired to a center-zero galvanometer; the bar magnet is inserted into and withdrawn from the coil along its axis.

★ Working note

Read the *peak* swing at the instant the magnet is moving fastest, and move the magnet along the *same marked distance* every run so only the time Δt changes — the flux change $\Delta\Phi_B$ stays the same.

From observation to model

Magnetic flux measures the field through an area

Draw a field as a family of lines and let their crowding stand for its strength, and a second and larger question becomes easy to put: not how strong the field is at one point, but how much of it a whole loop encloses. That total is what the **magnetic flux** Φ_B (phi) counts — how densely the lines run, multiplied by how much area they run through. Two loops sitting in one and the same field can enclose wildly different totals, and a single loop can enclose different totals as it is tilted, because a slanted

loop presents a smaller face to the lines. Only a loop turned squarely to the field catches the whole of *B*, and that is the case assumed everywhere in this lesson. Its unit follows from the product: one tesla spread across one square meter.

A changing flux induces an electromotive force

Your Explore runs separate the two things that might have mattered. Holding the magnet still, whether pressed against the coil or far from it, gave nothing at all — so the amount of flux threading the coil cannot be what drives the needle. Sweeping the same magnet across the same marked distance twice as fast delivered the identical change in half the time, and the swing doubled. What the coil answers to, then, is the *rate*: the emf tracks $\Delta\Phi_B/\Delta t$. Every turn is another loop enclosing that same change, so *N* turns multiply the result *N*-fold — the reason a bench coil is wound with hundreds. When the loop is rigid, only the field is free to vary, so the change in flux is evaluated from the change in *B* alone. That leaves the sign, and it is not decoration. Suppose the induced current reinforced the very change that produced it: the change would then feed itself, and current would grow with nothing supplying the energy. It must therefore fight the change instead, and that is the content of the minus sign — stated as a working rule, **Lenz's law**. Take the magnitude when only the size is wanted, and settle the direction from that rule.

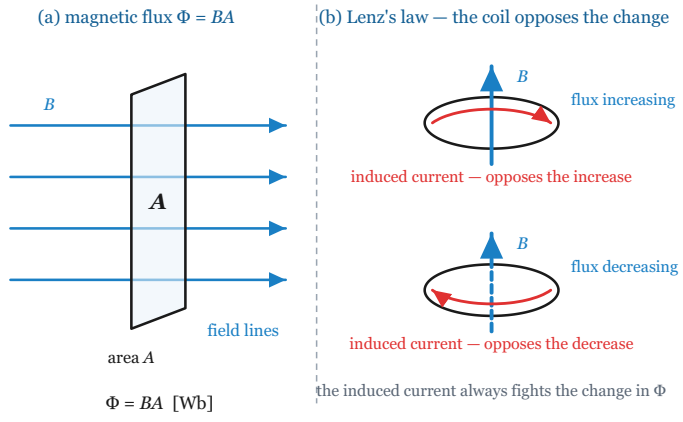
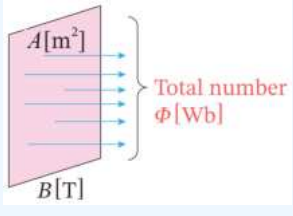


Figure 4-4-3 · Left: field lines of density B through an area A give the flux Φ = BA. Right: by Lenz's law the induced current circulates so its own flux opposes the change — resisting an increase (top) and a decrease (bottom).

Point

- Lines representing the state of magnetic flux density are called magnetic field lines. In a uniform magnetic field with a magnetic flux density B[T], B magnetic field lines pass through an area of 1 m² perpendicular to the field.
  The total number of magnetic field lines passing through a region with a magnetic flux density B[T] and an area A[m²] is called the **magnetic flux Φ_B**. The unit used is the **weber** (symbol **Wb**).
  Φ_B = **BA**



- The phenomenon in which a voltage is generated in a coil due to a change in the magnetic field inside the coil is called **electromagnetic induction**. The generated voltage is called **induced electromotive force**, and the current that flows as a result is called **induced current**.
- The induced electromotive force is generated in such a direction that the magnetic flux created by the induced current opposes the change in the externally applied magnetic flux.

★ **Term**

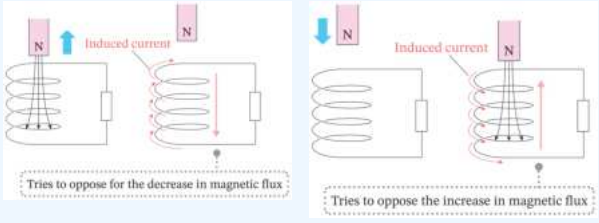
Magnetic flux — writing it as one product presumes two things at once: that the field is the same everywhere across the loop, and that the loop faces it square-on. Tilt the loop and only its projection counts; let the field vary from place to place and the single product has to give way to a sum over small patches, each with its own value of *B*.
Eddy current — a looping induced current set up inside a bulk conductor by a changing magnetic field, as in an IH heater.

★ **Key fact**

The scale of it: 500 turns of 2 cm² cross-section, sitting in a field that swings by 1 T in a tenth of a second, deliver about 1 V. That is why a generator is built from many turns spun quickly rather than from one enormous magnet — the rate is far cheaper to buy than the field.

★ **Note**

Two deltas, two separate mistakes. The first is to substitute the flux where its *change* belongs: a coil resting in the strongest steady field in the room induces nothing whatever. The second is to use the time you watched for rather than the time the change took — a magnet that falls through a coil in 0.05 s and then lies still for a minute is a 0.05 s event, and dividing by 60 s understates the emf a thousandfold.



When the magnetic flux Φ_B inside a coil with N turns changes to $\Phi_B+\Delta\Phi_B$ [Wb] after Δt seconds, the induced electromotive force ε[V] is

$$\varepsilon = \frac{-N\Delta\Phi_B}{\Delta t}.$$

The change in magnetic flux, $\Delta\Phi_B$, is determined by $\Delta\Phi_B = \Delta B \cdot A$.

However, when determining the magnitude of the induced electromotive force, use $\varepsilon = |-N\Delta\Phi_B/\Delta t|$.

Equation box

Magnetic flux	$\Phi_B = BA$ [Wb], with B in T and A in m^2
Faraday's law	$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$ (magnitude: $\varepsilon = \left\|-N\frac{\Delta\Phi_B}{\Delta t}\right\|$)
Change in flux	$\Delta\Phi_B = \Delta B \cdot A$ (uniform field, fixed area)

Worked Example

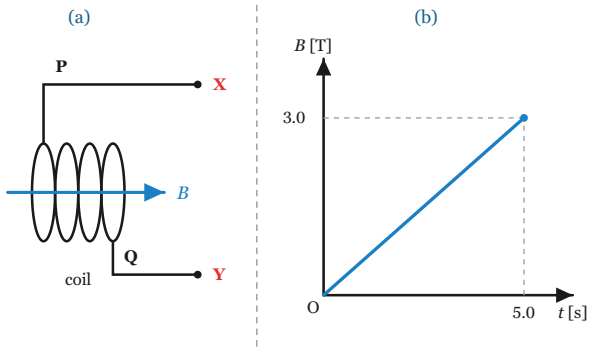


Figure 4-4-4 · (a) The coil with terminals P and Q, external points X and Y, and the reference arrow B that fixes the positive field direction. (b) The flux density B rises steadily from O to 3.0 T at t = 5.0 s.

Situation

The magnetic flux density B [T] inside a coil with 500 turns and a cross-sectional area of 2.0×10^{-4} m^2 changes as shown in Figure 4-4-4b, for the setup shown in Figure 4-4-4a. The magnetic flux density inside the coil is uniform, and the direction of the arrow in Figure 4-4-4a is taken as positive.

(a) Determine the magnitude of the induced electromotive force generated between P and Q of the coil in V.

(b) When a resistor is connected between P and Q, determine whether the current flows toward X or Y in Figure 4-4-4a.

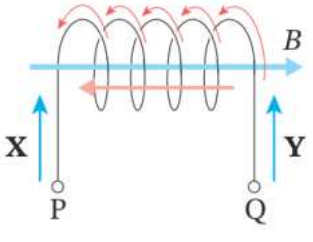


Figure 4-4-5 · Worked Example (b): the induced current in the coil, drawn in red, sets up a field opposing the growth of B, so the current in the outside circuit runs toward Y.

Given and required	N = 500 turns, A = 2.0×10^{-4} m². From Figure 4-4-4b the flux density rises steadily in the positive direction by ΔB = 3.0 T over Δt = 5.0 s. (a) The size of the emf ε [V] appearing across P and Q. (b) Which way its current runs round the circuit — toward X or toward Y.
Representation	See Figure 4-4-4: panel (a) shows the coil with terminals P and Q and the reference arrow B pointing between X and Y; panel (b) is the straight-line graph of B against t rising from O to 3.0 T at 5.0 s. Figure 4-4-5 carries the answer to (b): the induced current drawn on the same coil.
Strategy	Read ΔB and Δt straight off the graph, turn them into a flux change with the coil's own area, and put that rate through Faraday's law for the size. Settle the direction by a separate question: which way would a current have to circulate for its own field to fight the growth the graph shows?
Annotated solution	**(a)** Let ε be the size asked for. $\Delta\Phi_B = \Delta B \cdot A = 3.0 \times (2.0\times10^{-4}) = 6.0\times10^{-4}$ Wb. $\varepsilon = \lvert -N\,\Delta\Phi_B/\Delta t\rvert = \lvert -500 \times (6.0\times10^{-4})/5.0\rvert =$ **6.0×10^{-2} V**. **(b)** The graph rises, so the field in the positive sense is growing. The coil answers with a circulation whose own field points the other way, to the left, and that sense is **Y**.
Reasonableness check	Rewriting, $\varepsilon = N A\cdot(\Delta B/\Delta t) = 500 \times 2.0\times10^{-4} \times (3.0/5.0) = 0.10 \times 0.60 = 0.060$ V, which agrees with 6.0×10^{-2} V. Y is the sense whose own field pushes back against the growth; the other sense would feed it, and energy would appear from nowhere.
Explain it yourself	In one sentence, say why the answer to (b) would reverse if the graph in Figure 4-4-4b sloped downward instead of upward.

✍ **Now you try:** A coil of 200 turns and area 3.0×10^{-4} m² sits in a uniform field whose density rises steadily by 2.0 T in 4.0 s. Find the magnitude of the induced electromotive force.

Try

1. The magnetic flux density B [T] inside a coil with 100 turns and a cross-sectional area of 2.0×10^{-4} m² rises steadily by 8.0 T over 4.0 s, for the setup shown in Figure 4-4-6. The magnetic flux density inside the coil is uniform, and the direction of the arrow in Figure 4-4-6 is taken as positive.
   (a) Determine the magnitude of the induced electromotive force generated across the coil in V.
   (b) When a resistor is connected between P and Q, determine whether the current flows in the direction of X or Y in Figure 4-4-6.

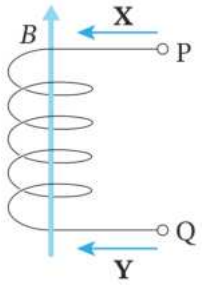


Figure 4-4-6 · Try: the coil with terminals P and Q, the reference arrow B fixing the positive direction, and the two senses X and Y in the outside circuit.

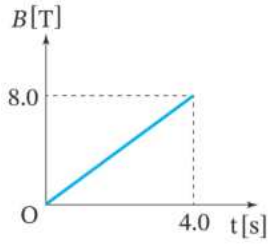


Figure 4-4-7 · Try: the flux density inside the coil rises steadily from O to 8.0 T at t = 4.0 s.

In a new context

This same law runs every electric generator. In a bicycle dynamo or a power-station alternator, a coil is spun in a magnetic field (or a magnet is spun past a fixed coil), so the flux through the coil rises and falls continuously. By $\varepsilon = -N\,\Delta\Phi_B/\Delta t$, a faster spin or more turns gives a larger electromotive force — the very reason generators use many-turn coils and rotate quickly.

➤ **Your turn:** A generator coil of 200 turns and area 5.0×10^{-4} m² sits in a field that changes steadily from 0 to 0.40 T in 0.10 s. Find the magnitude of the induced electromotive force.

Figure 4-4-8 · A bicycle dynamo: a spinning magnet changes the flux through a coil, inducing an emf that lights the lamp.

Nature of Science

From the original — IH cooking heaters and eddy currents. An "IH cooking heater," which can boil water without using a flame, uses electromagnetic induction. When the current in the internal coil is changed rapidly to change the magnetic field, an **eddy current** is induced in the bottom of the metal pot placed on top. The pot heats up through the **Joule heat** released as this current passes through the electrical resistance of the pot itself.

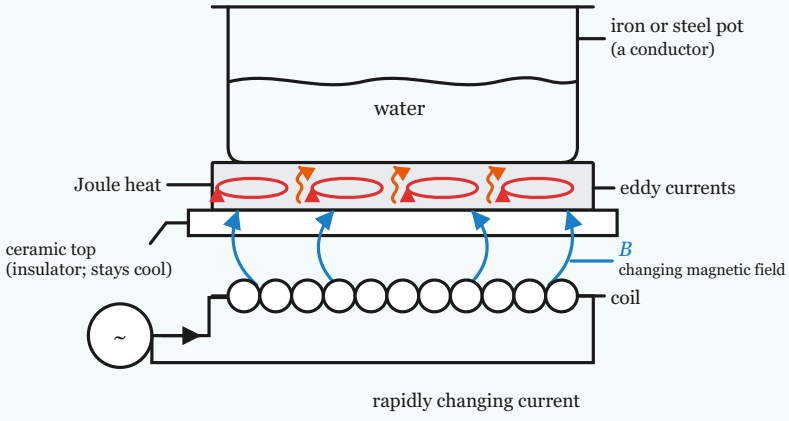


Figure 4-4-9 · Inside an IH cooking heater. A rapidly changing current in the coil under the ceramic top sends a changing magnetic field up through it and into the base of the pot. Because that base conducts, the changing flux drives eddy currents round inside the metal, and their Joule heating warms the pot itself — the ceramic top stays cool.

The same principle that lights a galvanometer also cooks food: a rapidly changing flux drives real currents in nearby conductors. Scientists trace one law — Faraday's — from a laboratory demonstration to an everyday appliance, which is how a physical model earns trust by explaining many separate situations at once.

Your turn: Suggest why an IH heater works with an iron or steel pot but fails to heat a glass or ceramic pot placed on the same surface.

Lesson question, answered — A current flows because moving the magnet changes the magnetic flux through the coil, and a changing flux induces an electromotive force even with no battery present. By Faraday's law the induced electromotive force is $\varepsilon = -N\,\Delta\Phi_B/\Delta t$, so it appears only while the flux is changing and grows with the rate of change; by Lenz's law its direction always opposes that change.

The scene we opened with.

Exercise

1. The magnetic flux passing through a 200-turn coil changed by 2.5×10^{-4} Wb in 0.20 seconds. Determine the magnitude of the induced electromotive force generated across the coil in V.
2. The magnetic flux density inside a coil with 100 turns and a cross-sectional area of 4.0×10^{-4} m² changed by 0.60 T at a constant rate in 4.0×10^{-2} seconds. Determine the magnitude of the induced electromotive force generated across the coil in V.
3. The magnetic flux density B [T] inside a coil with 200 turns and a cross-sectional area of 1.2×10^{-3} m² rises steadily by 4.0 T over 0.50 s, for the setup shown in Figure 4-4-10. The magnetic flux density inside the coil is uniform, and the direction of the arrow in Figure 4-4-10 is taken as positive.
   (i) Determine the magnitude of the induced electromotive force generated between P and Q of the coil in V.
   (ii) When a resistor is connected between P and Q, determine whether the current flows in the direction of X or Y in Figure 4-4-10.

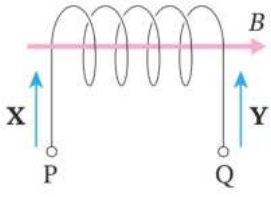


Figure 4-4-10 · Exercise 3: the coil with terminals P and Q, the reference arrow B, and the senses X and Y in the outside circuit.

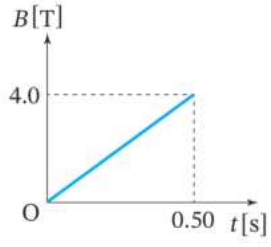


Figure 4-4-11 · Exercise 3: the flux density inside the coil rises steadily from O to 4.0 T at t = 0.50 s.

4-5 Self-Induction

❓ **Lesson question:** Why is it that even if you try to suddenly stop the current flowing through a coil, it does not stop immediately?

You will learn to

1. Explain why a coil fights every attempt to alter the current through it, and work out how large a back electromotive force (back emf) a given rate of change produces.
2. Read a switching problem at its two easy instants — the moment the switch moves, and long afterwards — and give the current and the back emf at each.
3. Work out how much energy a carrying coil is holding, and where that energy goes when the circuit is broken.

🔍 The phenomenon — predict before you continue

★ **Key idea**
Self-induction — the appearance of an induced emf in a coil caused by the change of the coil's own current.

Switch off an electric fan or drill at the wall and you may catch a tiny blue spark snapping across the contacts. The motor's coils were carrying a steady current, and the instant the switch breaks it, the coils fight the sudden change — inducing a large voltage that shoves the current on for a moment, jumping the widening gap as a spark. Switch on again and the current does not leap to full at once; it climbs. A coil resists every change to its own current, both when it starts and when it stops.

Figure 4-5-1 · Switching off a coil-carrying circuit: as the current is cut, the coil induces a large voltage that sparks across the opening contacts.

Predict

(a) When you close the switch, does the current in the coil reach its final value instantly, or build up gradually?

(b) The instant you open the switch, is the emf the coil induces small, or large enough to make a spark?

🔍 Explore — The Switch-Off Flash

What you'll do: cut a steady coil current at different speeds and read the neon flash it triggers, revealing that the induced emf grows with the rate of change of the current.

Time: 15 min

You will need:

- a coil of many turns
- a low-voltage battery (a few volts)
- a switch
- a neon lamp (strikes at about 90 V)
- an ammeter to read the steady current before the switch is opened

coil

Battery

switch

neon lamp

Steps

1. Connect a coil of many turns in series with a low-voltage battery and a switch, and wire a neon lamp across the coil; the lamp only strikes above about 90 V, far more than the battery supplies.
2. Close the switch, let a steady current I settle, and confirm the lamp stays dark.
3. Open the switch with a quick snap to cut the current; estimate the cut-off time Δt and note how bright the flash is.
4. Repeat, opening the switch more slowly (larger Δt); for each run work out the rate of fall $\Delta I/\Delta t = I/\Delta t$ and enter it in the last column.

Figure 4-5-2 · A neon lamp is wired across the coil; with a steady current the lamp stays dark, since the small battery voltage is far below its striking voltage.

Run	How you opened the switch	How fast the break was (slow / fast)	Flash brightness (dark / dim / bright)	Relative rate of fall (small / large)
1	quick snap			
2	medium			
3	slow release			

> ★ **Working note**
> The flash is very brief, so watch the lamp in a dimmed room. Open the switch with a clean snap to cut the current fast, and trust the trend across runs more than any single flash.

Analyze what you collected:

(a) Look down the last column: as the rate of fall $\Delta I/\Delta t$ grows, what happens to the flash brightness? What does that tell you about the size of the induced emf?

(b) The neon needs about 90 V to strike, yet the battery gives only a few volts — predict how large the induced emf must be at the instant of a bright flash, and relate your answer to $\varepsilon_L = |-L\,\Delta I/\Delta t|$.

✎ Write your explanation in two lines before you read the next page.

From observation to model

A coil opposes any change in its own current

The lamp told you where to look. A steady current left it dark, although that current was flowing the whole time; only the instant of the break lit it, and the runs whose last column — the rate of fall — was largest were the runs that flashed brightest. So the coil answers not to a current but to a *changing* current, and it answers with a voltage far above the cell that was driving it, since the neon will not strike below

about 90 V. Nothing in that goes beyond the previous lesson; the loop has simply been closed on itself. A coil encircles the very field it makes, so raising the current through it raises the flux threading it, and by Faraday's law a rising flux produces a voltage — this time in the very winding that caused it. Lenz's argument then fixes which way that voltage points: it must fight whatever the current is doing, holding it back while it climbs and propping it up while it dies away. Hence the name given to the effect, and hence the two panels beside the Point: the same coil, once resisting a rise and once resisting a fall.

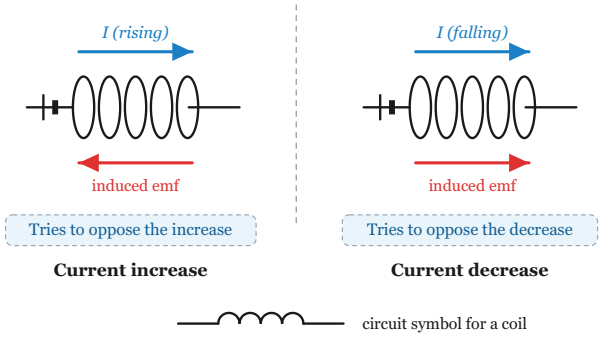


Figure 4-5-3 · When the current rises the coil induces an emf that opposes the increase; when it falls the coil induces an emf that opposes the decrease. Alongside is the circuit symbol for a coil.

★ **Key fact**

The scale of it: cut 3 A out of a 0.10 H coil in a thousandth of a second and about 300 V appears across it — a hundred times the cell that was driving it, and easily enough to jump the widening gap in an opening switch. Slow the same cut to a tenth of a second and it falls to 3 V, and no spark is seen.

How big is the induced emf?

Follow the chain: the flux a coil makes is proportional to its own current, and the voltage is proportional to the rate at which that flux changes. Two proportionalities in a row collapse into one, so the voltage can be read off the rate of change of the current alone, with a single constant between them. That constant bundles up everything geometric about the coil — how many turns, how tightly they are wound, what core they sit on — into one number, which is why a data sheet quotes it and never asks you about the winding. The unit is a large one: a coil worth one **henry** answers a current sliding by one ampere every second with one volt. The negative sign is only bookkeeping for the direction already argued; take the size when the size is what is wanted.

A coil in a DC circuit

Two instants can be read off without solving anything that varies in time. At the first of them, the moment the switch moves, the coil cannot yet have altered what it carries: an abrupt jump would mean an unlimited rate of change and an unlimited back emf, which no circuit can supply. So carry the previous value straight across the switching. At the second, long afterwards, the current has stopped changing at all; the rate falls to zero, the back emf goes with it, and the winding is left as a length of wire like any other. Between the two the current slides gradually from one value to the other, and that slide is the curve drawn beside the Point.

★ **Term**

Self-inductance — the model treats L as a fixed property of the coil, which is true enough for an air-cored winding. A coil on an iron core is another matter: past a certain current the core saturates, L falls away, and the back emf stops following the rate of change in a straight line. The number on the data sheet then holds only below that current.

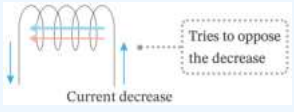
Energy stored in a coil

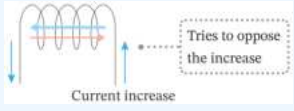
Getting the current up there in the first place cost work, because every increment had to be forced through against the coil's own opposing voltage. That work is not lost as heat — it is held in the field the winding has built, and it is handed back the instant the current collapses, which is precisely where the energy in the switch-off flash comes from. Add up the work done while the current climbs from zero. At the instant the current stands at some value i, the coil is pushing back with a voltage of size $L\Delta i/\Delta t$, so forcing the next increment Δi through in that time costs $Li\Delta i$ — a price that is not fixed but rises in step with the current already established. Plot that opposition against the current and it is a straight line out of the origin, standing at LI by the time the final current I is reached; the work done is the area beneath it, a triangle of base I and height LI. So the store is $\frac{1}{2}LI^2$. The $\frac{1}{2}$ is not a factor to be memorized: it is the average of an opposition that began at nothing and ended at LI.

It is also why the store grows with the *square* — double the current and both the base and the height of that triangle double, giving four times the area.

Point

The phenomenon where an induced electromotive force is generated in a coil in a direction that opposes the change in current when the current I [A] flowing through the coil changes to $I+\Delta I$ [A] after Δt seconds is called **self-induction**. The magnitude of the induced electromotive force ε_L [V] at this time is,

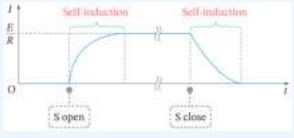


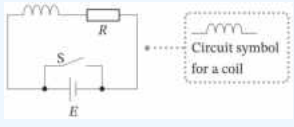


$\varepsilon_L = \left|\frac{-L\Delta I}{\Delta t}\right|$

This proportionality constant L is called **self-inductance**, and its unit is the **henry** (symbol **H**).

Concept of DC circuits including coils





Immediately after operating the switch...
The flowing current is the same as before the operation.
When enough time has passed after operating the switch...
The coil can be considered a conducting wire.
Example:
Immediately after opening S : $I = 0$ (Same as immediately before opening)
When enough time has passed after opening S : $I = \varepsilon/R$ (Consider the coil as a conducting wire)
Immediately after closing S : $I = \varepsilon/R$ (Same as immediately before closing)

When a current I [A] flows through a coil with self-inductance L [H], the energy U [J] stored in the coil is
$U =$ **$\frac{1}{2}LI^2$**.

Equation box

Induced emf	$\varepsilon_L = \left\|-L\frac{\Delta I}{\Delta t}\right\|$ (unit: V; L in henries [H], $\frac{\Delta I}{\Delta t}$ = rate of change of current)
Energy stored	$U = \frac{1}{2}LI^2$ (unit: J)
DC coil rules	Immediately after switching: I unchanged $\Longrightarrow$ after a long time: coil = conducting wire ($\varepsilon_L = 0$)

★ Note

The continuity applies to the branch with the winding in it, and to no other: a resistor branch elsewhere may well change its current the instant the switch moves. And "long enough" means long compared with the settling of this particular circuit, which for a large coil and a small resistance can be far longer than the eye suggests.

🛠 Worked Example

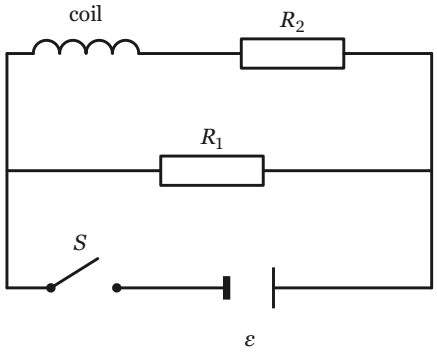


Figure 4-5-4 · Battery of emf ε: resistor R_1 sits directly across the battery, while resistor R_2 in series with the coil forms a second branch; switch S in the supply line.

Situation	As shown in the figure on the right, a battery with an electromotive force ε [V], resistors with resistance values R_1 and R_2 [Ω], a coil, and a switch are connected. For the following cases, determine the magnitude of the induced electromotive force ε_L [V] generated in the coil and the current I [A] flowing through the R_1 resistor. (a) Immediately after closing the switch. (b) When enough time has passed after closing the switch.
Given and required	Circuit: R_1 is connected directly across the battery ε; the coil and R_2 form a second branch across the battery. Required in each case: the induced emf ε_L across the coil and the current I through R_1.
Representation	See Figure 4-5-4: battery ε, resistor R_1 across the battery, resistor R_2 in series with the coil in the other branch, switch S. Figure 4-5-5 traces the current that answers (a).
Strategy	Apply the two DC-coil rules. **Immediately after** closing the switch, the coil holds the current it carried before (zero), so no current flows in the R_2–coil branch and the full battery emf appears across the coil. **After a long time**, the current is steady, so the coil acts as a plain wire and its induced emf is zero. In both cases R_1 sits directly across the battery, so the current through it is ε/R_1.

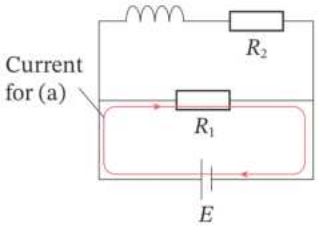


Figure 4-5-5 · Worked Example (a): immediately after the switch closes the coil branch carries nothing, so the whole circulation is the loop drawn in red — battery, R_1, and back.

Annotated solution	**(a) Immediately after closing.** The current in the coil was zero and cannot jump, so the coil branch carries no current and there is no drop across R_2; the whole battery emf appears across the coil: ε_L = **ε [V]**. Since R_1 is directly across the battery, the current through it is I = **ε/R_1 [A]**. **(b) After a long time.** The current is steady, so $\Delta I/\Delta t = 0$ and the coil is a plain wire: ε_L = **0 V**. R_1 is still directly across the battery, so I = **ε/R_1 [A]**.
Reasonableness check	Both limits make sense: at the first instant the coil blocks its branch and behaves like an open gap, so it must carry the full emf; long after, it offers no opposition and drops no voltage. The current in R_1 never depends on the coil branch because R_1 is wired straight across the battery.
🧐 **Explain it yourself**	In one sentence, explain why the induced emf across the coil is largest at the very first instant after the switch is closed and falls to zero later.

✍️ **Now you try:** A current of 0.40 A flows through a coil of self-inductance 5.0 H. Determine the energy stored in the coil in J.

🔆 Try

1. As shown in the figure on the right, a battery with an electromotive force ε [V], resistors with resistance values R_1 and R_2 [Ω], a coil, and a switch are connected. For the following cases, determine the magnitude of the induced electromotive force ε_L [V] generated in the coil and the current I [A] flowing through the R_1 resistor. (i) Immediately after closing the switch. (ii) When enough time has passed after closing the switch.
2. The current flowing through a coil with a self-inductance of 10 H increased from 0.20 A to 0.30 A. By how many J did the energy stored in the coil increase?

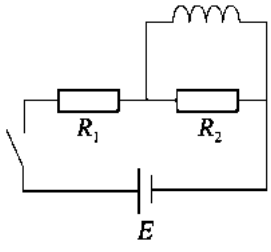


Figure 4-5-6 · Try 1: the battery ε, the resistors R_1 and R_2, the coil, and the switch, connected as described.

🌐 In a new context

A gasoline engine uses this effect on purpose. Its ignition coil carries a steady current in a **primary** winding; when a contact suddenly breaks that current, $\Delta I/\Delta t$ is enormous, so the self-induced emf $\varepsilon_L = |-L\,\Delta I/\Delta t|$ across that winding leaps to several hundred volts. The coil is wound as a transformer, with far more turns in its **secondary** than its primary, so the collapsing field induces tens of thousands of volts there — enough to jump a spark across the plug and ignite the fuel. The effect that snaps a spark across an opening switch is thus harnessed, and then stepped up, to fire the engine.

➤ **Your turn:** In a coil of self-inductance 8.0 H, a current of 3.0 A is cut to zero in 1.0×10^{-3} s. Determine the magnitude of the induced emf in V.

Nature of Science

From the original — lightning surges and self-induction. During a lightning strike, a sudden large current (a lightning surge) can flow through power lines. Self-induction then occurs in the coils inside home appliances and can generate an extremely large back emf of thousands of volts. Because this high voltage can destroy the circuits of electronic devices such as computers, engineers guard against it using power strips fitted with surge protectors.

Here the same law explains both a wanted effect (an ignition spark) and an unwanted hazard (a surge that wrecks a computer). Recognizing that one relation, $\varepsilon_L = |-L\,\Delta I/\Delta t|$, governs a benchtop coil, a car engine, and a lightning surge is how physicists turn a single principle into both a device and a safeguard.

Joseph Henry reported and correctly interpreted this effect in July 1832, working in America independently of Faraday in London, and the unit of inductance carries his name for it. Two people can reach the same result apart, and priority is settled by the record rather than by reputation.

Your turn: Suggest one design feature of a surge protector that could be tested to check whether it really limits the back emf reaching a connected computer.

Figure 4-5-7 · A cutaway automotive ignition coil, with a few thick primary turns over many fine secondary turns on an iron core, sends high voltage through a single insulated lead to a spark plug, where a blue-white spark jumps the electrode gap and ignites a flame kernel at the instant the breaker contacts in the primary circuit spring apart.

Figure 4-5-8 · A lightning surge travels along the power line into a home; a surge-protector power strip stands between the outlet and the computer, while inside an appliance the coil builds a large back emf.

The scene we opened with.

Lesson question, answered — A coil resists any change in its own current: when the current tries to change, the coil induces an emf $\varepsilon_L = |-L\,\Delta I/\Delta t|$ that opposes the change. Cutting the current fast makes $\Delta I/\Delta t$ huge, so the induced emf is very large — large enough to keep the current flowing for a moment and to spark across the opening contacts. That is why the current in a coil does not stop immediately.

Exercise

1. The current flowing through a coil with a self-inductance of 10 H increased by 0.60 A at a constant rate in 0.20 seconds. Determine the magnitude of the induced electromotive force generated in the coil in V.
2. When the current flowing through a coil increased at a rate of 2.0 A per second, an induced electromotive force of 5.0 V is generated in the coil. Determine the self-inductance of the coil in H.
3. As shown in the figure on the right, a battery with an electromotive force of 20 V, a resistor with a resistance of 40 Ω, a coil with a self-inductance of 0.20 H, and a switch are connected. (i) Determine the current flowing in the circuit immediately after closing the switch in A. (ii) When enough time has passed after closing the switch, determine the current flowing in the circuit in A. (iii) At the time of (ii), determine the energy stored in the coil in J.

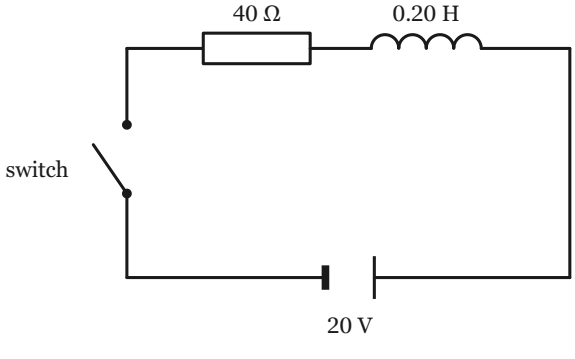
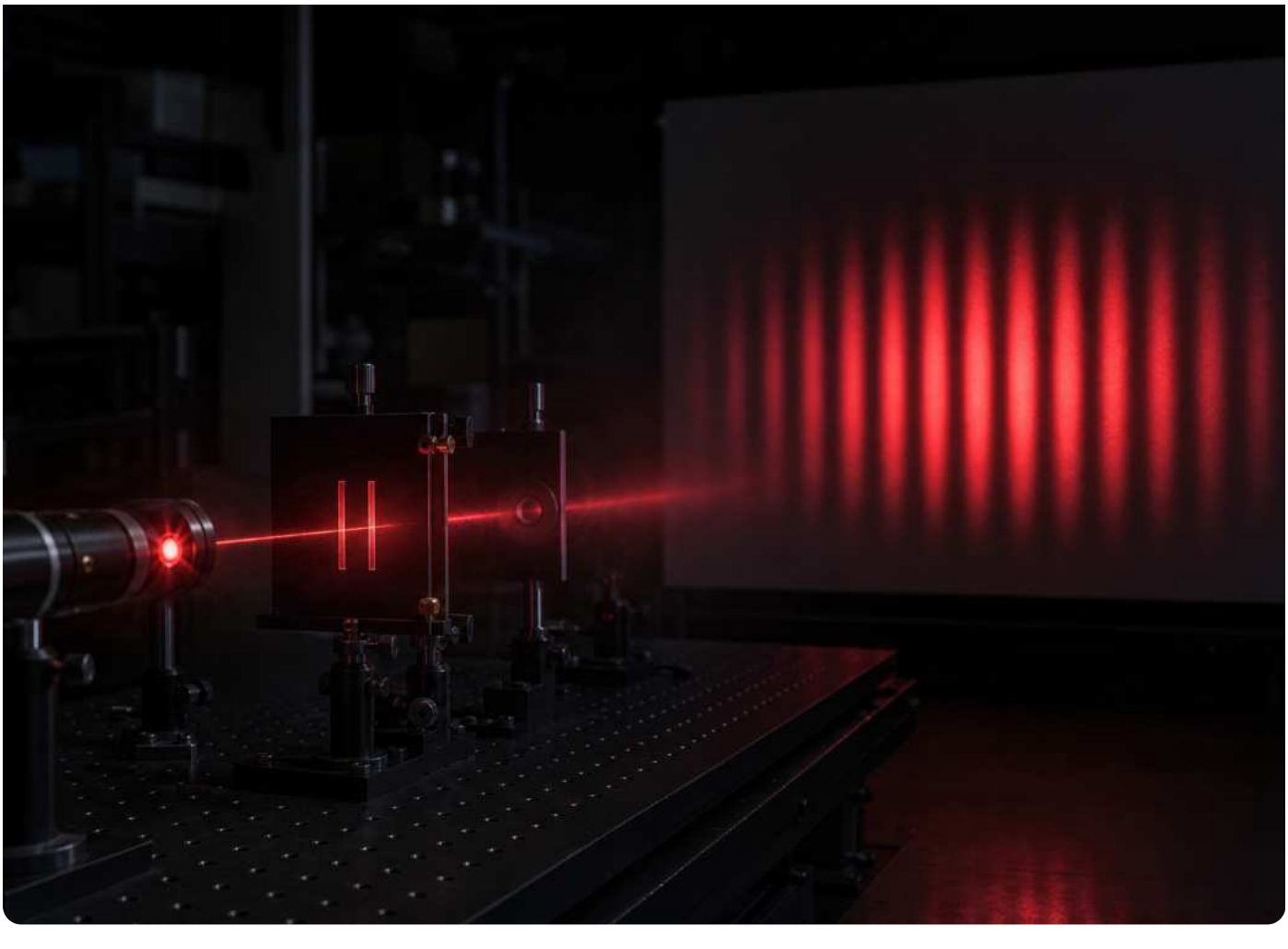


Figure 4-5-9 · A series loop of a 20 V battery, a 40 Ω resistor, a 0.20 H coil, and a switch.



UNIT 5

The Quantum Nature of Light and Matter

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5-1 Particle Nature of Light

❓ **Lesson question:** What phenomena can be explained if we consider light not as a wave but as a collection of particles?

You will learn to

1. Work out how much energy one packet of light carries, starting either from the color of the light or from its wavelength.
2. Predict what a named metal does under a given lamp: whether any charge leaves it at all, how fast the quickest electrons go, and what changes if the lamp is made brighter rather than bluer.
3. Measure two constants off one experimental line — a universal one from its gradient and a metal-specific one from where it cuts the axes.

🔍 The phenomenon — predict before you continue

★ **Key idea**
Photon — the indivisible packet in which light is handed over. What sets its energy: the color of the light, or its brightness?

A whole afternoon under a bright indoor lamp never burns your skin, yet a short spell in the sun can — because sunlight also carries ultraviolet. Turning the lamp brighter still does nothing, but the higher-frequency ultraviolet harms skin even in far smaller amounts. What a beam of light can do seems to hinge on its color (its frequency), not on how bright it is — the first clue that light hands over its energy in fixed packets.

Figure 5-1-1 · An ordinary bright lamp leaves skin unharmed, while far weaker ultraviolet burns it — a clue that light's effect is set by its frequency (color), not its brightness.

Predict

(a) If you make the ordinary visible lamp far brighter, will it begin to harm skin the way a little ultraviolet does?

(b) If the effect is set by frequency rather than brightness, does dimming an ultraviolet source (fewer photons) change the energy each single photon delivers?

🔍 Explore — Color or brightness: what discharges the plate?

What you'll do: Charge a zinc-plate electroscope, then test filtered light of rising color (frequency) at different brightnesses to find what actually makes the leaf fall.

Time: 15 min

You will need:

- Gold-leaf electroscope with a clean zinc plate
- Lamp on an adjustable stand with a brightness control
- Set of color filters (red → violet) and an ultraviolet source
- Charged rod to give the plate a negative charge

electroscope

lamp on stand

color filter (one of a set)

charged rod

Steps

1. Give the zinc plate a negative charge with the charged rod until the gold leaf rises, and record the leaf angle.
2. Fit the red filter, aim the lamp at the plate, and raise the brightness in steps — record in the last column whether the leaf falls.
3. Swap the filters toward violet and finally to the ultraviolet source; for each color, working down the table from low to high frequency, note whether the leaf discharges.
4. For the first color that discharges the plate, dim the lamp and check whether the leaf still falls.

Run	Filter color (frequency ν rising)	Lamp brightness	Leaf discharges? (Y / N)
1			
2			
3			

Figure 5-1-2 · Bench setup: a lamp on a stand with a brightness control and a swappable visible/ultraviolet source, aimed at the zinc plate of a gold-leaf electroscope.

> ★ **Working note**
> A falling leaf means charge is leaving the plate — watch the leaf, not the lamp. Keep the plate the same distance from the lamp for every run, so only the color and the brightness change between rows.

Analyze what you collected:

(a) Reading down the column from red to ultraviolet, where does the leaf first start to fall — and does turning a failed color up to full brightness ever discharge the plate?

(b) Which single property of the light decides whether charge leaves the metal? Explain why this contradicts the wave idea that a brighter beam, carrying more energy, should always free charge.

✎ Write your explanation in two lines before you read the next page.

From observation to model

Light as a stream of photons

Why should light have to arrive in packets at all? The Explore result is the clue. However far the red lamp is turned up the leaf never falls, while a feeble ultraviolet source discharges the plate at once. A wave spreads its energy smoothly across the whole plate, so a stronger wave ought always to win in the end and color ought not

to matter; the measurement says the reverse. It fits only if the beam hands over its energy in indivisible lumps whose size is fixed by the color — turning the lamp up sends more lumps, never bigger ones. Measuring how the lump size grows with color gives a straight proportionality, and the constant that scales it comes out the same for every metal and every color ever tried, which is what makes it a constant of nature rather than a property of the lamp.

The photoelectric effect

Nothing in the wave picture forbids a slow start: a weak wave should simply need longer to gather enough energy at one spot. The bench refuses that too. Below a certain color the current is exactly zero however long one waits and however bright the lamp, and above it the current appears with no measurable delay even when the light is so faint that a wave would need hours to accumulate the energy. Two independent dials are therefore at work — one decides *whether* an electron gets out and how fast it leaves, the other decides only *how many* get out. That separation is only possible if a single packet deals with a single electron in one all-or-nothing transaction.

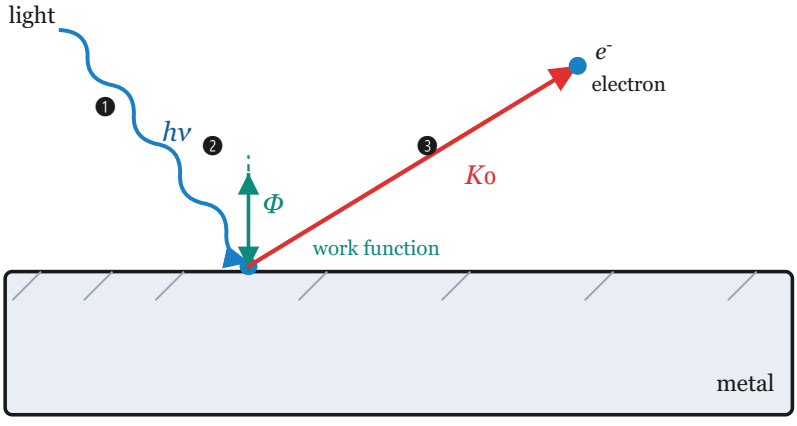


Figure 5-1-3 · The photon mechanism in three stages: ❶ one photon of energy h*ν* is absorbed by a single electron; ❷ the electron pays the work function Φ to escape the metal; ❸ the remainder leaves as the maximum kinetic energy K_0.

Follow the energy through one transaction. The packet arrives carrying $h\nu$; getting clear of the surface costs a fixed toll Φ that belongs to the metal; whatever survives the toll leaves as kinetic energy. Electrons that start deeper down pay extra on the way out, so the difference is a ceiling rather than a common value — which is what the subscript on K_0 records. Now read that ceiling as the equation of a straight line in the variable ν: it has the form $y = (h)x + (-\Phi)$, so plotting the electron energy against frequency must give a straight line of gradient h cutting the energy axis at $-\Phi$. Two constants of quite different character then drop out of one graph: the gradient gives h, identical for every metal, while the intercept gives Φ, different for each. The line crosses the frequency axis where the packet is just big enough to pay the toll and no more, $h\nu_0 = \Phi$, so

$$\nu_0 = \frac{\Phi}{h}$$

— below that point the equation would return a negative kinetic energy, which is nature's way of saying nothing comes out at all.

★ **Key fact**

Scale — Planck's constant is among the smallest numbers in physics, so one packet of visible light is minute: the Worked Example below values a single photon at 6.0×10^{-19} J. Ordinary light delivers so many packets per second that the graininess never shows, which is why the wave picture served for two centuries before anyone watched a single metal surface closely.

★ **Note**

Careful — “stronger light” and “higher-frequency light” are two different changes and questions mix them on purpose; read which one is being held fixed before answering. Note also that turning up the brightness does raise the current, so a plot of current against brightness climbs while a plot of electron speed against brightness stays flat.

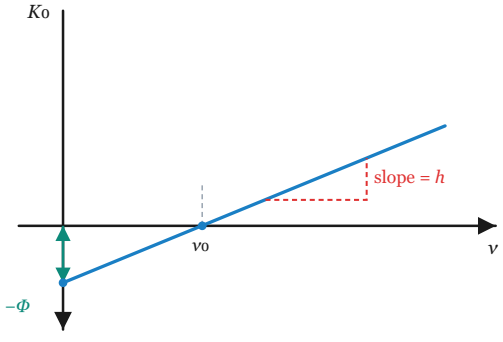


Figure 5-1-4 · Maximum kinetic energy K_0 against frequency ν: a straight line of slope h that meets the K_0-axis at the intercept $-\Phi$ and crosses the ν-axis at the threshold frequency ν_0.

Point

Photon Hypothesis

- Light is a stream of particles. This particle is called a **photon (light quantum)**. The intensity of light is determined by its frequency and the **number of photons**.
- The energy E [J] of a single photon is proportional to the frequency of light ν [Hz], and is expressed by the following formula.

$E = h\nu$ where h is a constant called **Planck's constant**, and $h \approx 6.63\times10^{-34}$ J·s.

The phenomenon in which electrons are emitted from a metal when light shines on its surface is called the **photoelectric effect**. These emitted electrons are called **photoelectrons**. The photoelectric effect does not occur if the frequency of the shining light is less than a certain value. When the **frequency** of light is increased, the kinetic energy of the photoelectrons increases. When light is made stronger (increasing the number of photons), the number of photoelectrons increases. (The kinetic energy of the photoelectrons does not change.)

Explanation of the photoelectric effect by the photon hypothesis

❶ One photon with energy $h\nu$ gives energy to one electron in the metal.

❷ The electron that gained energy pops out of the metal. (The minimum energy required to pop out of the metal, Φ, is called the work function)

❸ The remainder after subtracting the work function from the photon's energy becomes the maximum kinetic energy K_0 of the electron.

$K_0 = h\nu - \Phi$

Graph of light frequency ν and maximum kinetic energy of electrons K_0

Slope of the straight line h : **Planck's constant**

Intercept of the straight line $-\Phi$: **Work function** Φ

Intersection of the straight line and the ν-axis ν_0 :

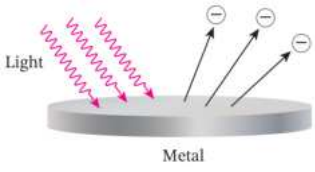
Threshold frequency ν_0

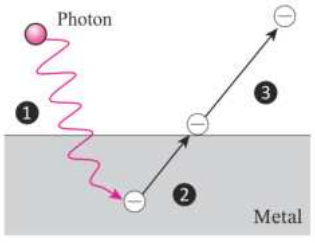
When $\nu < \nu_0$, electrons are not emitted

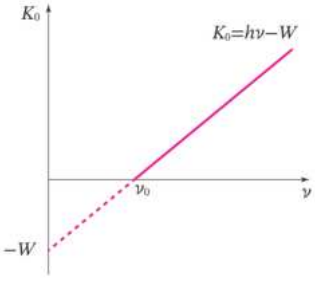
When $\nu > \nu_0$, electrons are emitted

★ Term

Where the model stops — Φ is a property of the metal surface, never of the light, so no setting of the lamp can shift it, while a film of oxide or a fingerprint on the plate shifts it a great deal. The photoelectric equation gives a ceiling, not every electron's speed: only electrons freed right at the surface reach it. And the model fixes energies, not timings — it says nothing about *when* a particular electron will leave.







📊 Equation box	
Photon energy	$E = h\nu$ ($h \approx 6.63\times10^{-34}$ J·s); from wavelength, $\nu = \frac{c}{\lambda}$ so $E = \frac{hc}{\lambda}$
Photoelectric equation	$K_0 = h\nu - \Phi$ (maximum kinetic energy of the photoelectrons; Φ is the work function)
Graph of K_0 vs ν	slope = h intercept = $-\Phi$ ν-axis crossing = ν_0, with $\Phi = h\nu_0$
Threshold	$\Phi = h\nu_0$

🛠 Worked Example

Situation	Light with a wavelength of 3.3×10^{-7} m is incident on a metal with a work function of 3.7×10^{-19} J. Answer to 2 significant figures, assuming Planck's constant is 6.6×10^{-34} J·s and the speed of light is 3.0×10^{8} m/s. (a) Determine the energy E of one photon of this light in J. (b) Determine the maximum kinetic energy K_0 of the photoelectrons popping out of the metal in J. (c) If the intensity of light is constant and only the wavelength of light is shortened, describe how the maximum kinetic energy of the photoelectrons changes.
Given and required	$\lambda = 3.3\times10^{-7}$ m, $\Phi = 3.7\times10^{-19}$ J, $h = 6.6\times10^{-34}$ J·s, $c = 3.0\times10^{8}$ m/s. Find (a) the photon energy E, (b) the maximum kinetic energy K_0, (c) how K_0 responds to a shorter wavelength.
Representation	See Figure 5-1-3 for the photon–electron energy transfer: the photon supplies $E = h\nu$, the work function Φ is paid to escape, and the remainder is K_0.
Strategy	Express the frequency in terms of wavelength and speed, $\nu = c/\lambda$, and use $E = h\nu$ to get the photon energy. Then subtract the work function from that photon energy. For part (c), use $\nu = c/\lambda$ to see how the frequency, and hence the electron energy, changes as λ shrinks.

Annotated solution	**(a)** Frequency from wavelength, $\nu = c/\lambda = 3.0\times10^{8} / 3.3\times10^{-7}$, then $E = h\nu$: $E = (6.6\times10^{-34}) \times (3.0\times10^{8} / 3.3\times10^{-7}) =$ **6.0×10^{-19} J**. **(b)** Subtract the work function from the photon energy: $K_0 = (6.0\times10^{-19}) - (3.7\times10^{-19}) =$ **2.3×10^{-19} J**. **(c)** As the wavelength becomes shorter, the frequency becomes larger, so the maximum kinetic energy of the photoelectrons becomes larger.
Reasonableness check	The photon energy 6.0×10^{-19} J is larger than the work function 3.7×10^{-19} J, so emission does occur, and the leftover $K_0 = 2.3\times10^{-19}$ J is positive and smaller than the photon energy — exactly as $K_0 = h\nu - \Phi$ requires.
Explain it yourself	In one sentence, say why making the same light brighter (without changing its wavelength) would not change the maximum kinetic energy of the photoelectrons.

✍️ **Now you try:** Light with a wavelength of 6.6×10^{-7} m is incident on a metal with a work function of 2.0×10^{-19} J ($h = 6.6\times10^{-34}$ J·s, $c = 3.0\times10^{8}$ m/s, 2 significant figures). Determine the photon energy E and the maximum kinetic energy K_0.

🔆 Try

1. Determine the following to 2 significant figures. When light with a frequency of 5.0×10^{14} Hz is incident on a metal with a work function of 3.0×10^{-19} J, the photoelectric effect occurs. Determine the maximum kinetic energy K_0 of the photoelectrons popping out of the metal in J. Assume Planck's constant is 6.6×10^{-34} J·s.
2. When light of various frequencies was shone on a certain metal, the relationship between the frequency of light ν [Hz] and the maximum kinetic energy K_0 [J] of the photoelectrons was as shown in Figure 5-1-5. Determine the work function of this metal in J. Assume Planck's constant is 6.6×10^{-34} J·s.

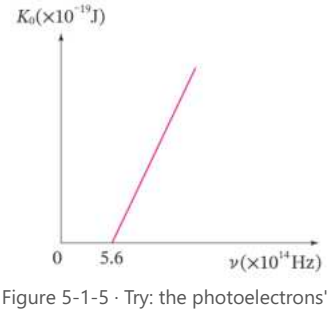


Figure 5-1-5 · Try: the photoelectrons' greatest energy plotted against the light frequency; the line meets the frequency axis at 5.6×10^{14} Hz.

🌐 In a new context

The same photon-in, electron-out mechanism runs light sensors everywhere: the photomultiplier in a radiation detector, the light meter in a camera, the automatic-door beam, and the photodiode in a barcode scanner all rely on light of high enough frequency freeing charge from a surface or across a junction. Because only frequency (not brightness) can lift an electron over the work function, each device is built with a material whose threshold frequency matches the light it must sense.

➤ **Your turn:** A metal has a work function of 2.0×10^{-19} J. Determine the threshold frequency ν_0 below which no photoelectrons are emitted ($h = 6.6\times10^{-34}$ J·s, 2 significant figures).

🧪 Nature of Science

From the original — solar cells and the photoelectric effect. Solar cells used in calculators and solar panels on roofs apply the photovoltaic effect. When particles of light (photons) hit a semiconductor and eject electrons, light energy is converted directly into electrical energy, generating an electric current.

The photon hypothesis earned its place by explaining exactly what the wave model could not: a frequency threshold for emission, and a maximum kinetic energy that depends on frequency rather than on brightness. A model in physics is kept when it accounts for the stubborn observations a rival model leaves unexplained, and then it is put to work — here, turning sunlight straight into current.

Hertz stumbled on the effect in 1887, and Lenard showed in 1902 that color rather than brightness decided whether charge left the metal — a result no wave account could absorb. Einstein's explanation of 1905 was the reason for his Nobel Prize in 1921, not relativity. An anomaly that refuses to go away is how physics is forced to change its mind.

Your turn: Suggest one measurement on a photoelectric cell that would let a scientist decide between the wave picture and the photon picture of light.

Lesson question, answered — Treating light as a collection of particles settles three things the wave picture cannot. First, that a threshold exists at all: a wave of any color should free charge in the end, whereas a packet either pays the surface toll or it does not. Second, that the electrons' speed tracks the color and ignores the brightness, because one electron deals with one packet. Third, that emission starts instantly even in dim light, because nothing has to be accumulated. The straight line obtained by plotting the electrons' energy against frequency is the quantitative signature of all three: its gradient measures a universal constant, its intercept measures the metal's own toll, and its crossing point marks where emission begins.

Figure 5-1-6 · An automatic sliding glass door fitted with a paired light emitter and photoelectric receiver low on the frames; a thin straight beam crosses the doorway and the approaching person's leg interrupts it, signaling the panels to slide open.

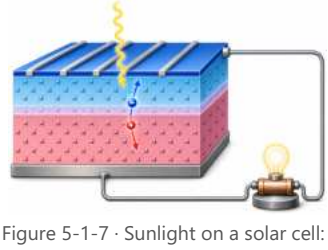


Figure 5-1-7 · Sunlight on a solar cell: photons strike the semiconductor and eject electrons across the junction, driving an electric current that powers a calculator.

The scene we opened with.

Exercise

1. Determine the following to 2 significant figures. When light with photon energy of 9.0×10^{-19} J is incident on a metal with a work function of 6.9×10^{-19} J, the photoelectric effect occurred. What is the maximum kinetic energy K_0 of the photoelectrons popping out of the metal in J?
2. Light with a wavelength of 2.2×10^{-7} m was shone on a metal with a work function of 4.4×10^{-19} J. Assume Planck's constant is 6.6×10^{-34} J·s and the speed of light is 3.0×10^{8} m/s.
   (i) Determine the energy E of one photon of this light in J.
   (ii) Determine the maximum kinetic energy K_0 of the photoelectrons popping out of the metal in J.
   (iii) If the wavelength is constant and only the intensity of light is increased, describe how the maximum kinetic energy of the photoelectrons changes.
3. When light of various frequencies is incident on a certain metal, the relationship between the frequency of light ν [Hz] and the maximum kinetic energy K_0 [J] of the photoelectrons emitted from the metal is as shown in Figure 5-1-8. Determine the work function of this metal in J. Assume Planck's constant is 6.6×10^{-34} J·s.

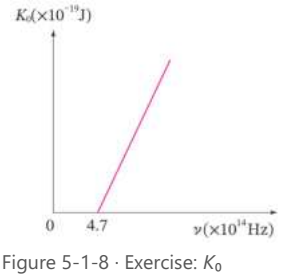


Figure 5-1-8 · Exercise: K_0 against ν for the metal in this problem; the line meets the frequency axis at 4.7×10^{14} Hz.

5-2 X-rays

❓ **Lesson question:** How are X-rays — the penetrating rays used to see through luggage and bodies alike — produced?

You will learn to

1. Account for the two parts of an X-ray spectrum — the broad hump and the sharp lines — and say which part moves when the tube voltage is changed and which moves when the target metal is changed.
2. Obtain the energy and the momentum of one X-ray photon from its wavelength, and give that momentum a direction.
3. Predict the short-wavelength cut-off of a tube from its accelerating voltage, and convert freely between joules and electron volts.

🔍 The phenomenon — predict before you continue

At an airport, every bag rolls through a scanner that shows its contents on a screen without ever being opened — the machine floods the bag with X-rays, penetrating rays it makes on the spot. To make them, electrons are accelerated through a high voltage and crashed into a metal target; most of their energy turns to heat, but a small fraction is flung out as X-rays, with wavelengths far shorter than ultraviolet. Raise the voltage and the rays come out more penetrating.

★ **Key idea**

X-rays — radiation that is manufactured rather than found: drive electrons through a voltage V into a metal target and part of their energy comes back out as rays far more penetrating than light. What sets their wavelengths?

Figure 5-2-1 · An airport scanner makes X-rays by driving accelerated electrons into a metal target; the rays pass through the bag to image its contents.

Predict

(a) If you raise the accelerating voltage, does the shortest wavelength of the emitted X-rays get longer or shorter?

(b) Do the sharp spikes move when you change the voltage, or only when you change the target metal?

🔍 Explore — Reading the X-ray spectrum

What you'll do: read the shortest wavelength off two measured X-ray spectra taken at different accelerating voltages, and test whether the product $V \times \lambda_{min}$ stays the same.

Time: 10 min

You will need:

- The two measured X-ray spectra below
- A ruler to read the cut-off
- A calculator

Ruler

Tube supply (given *V*)

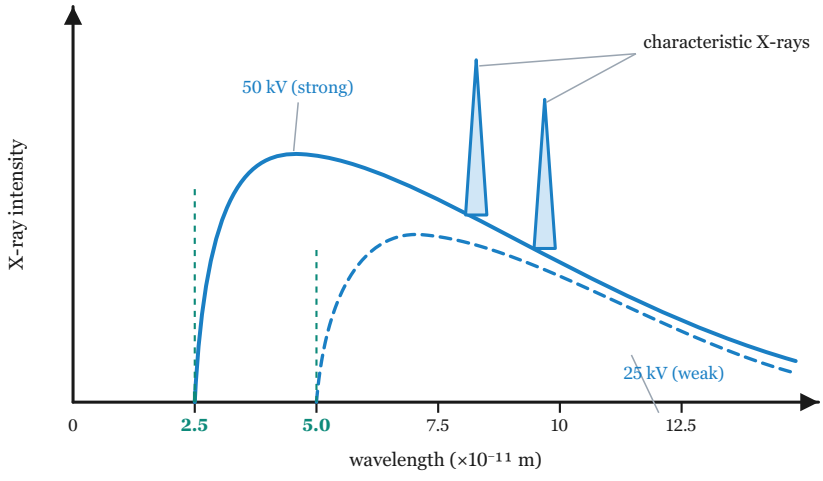


Figure 5-2-2 · Two measured X-ray spectra from the same metal target: the solid curve at 50 kV cuts off at 2.5×10^{-11} m, the dashed curve at 25 kV at 5.0×10^{-11} m. The characteristic peaks sit at wavelengths fixed by the metal, so they do not move with the voltage.

> ★ **Working note**
> Read λ_{min} where a continuous curve meets the wavelength axis in a sharp cut-off — not at a spike; the tall characteristic peaks are fixed by the metal, so ignore them when you locate the cut-off. Use the same wavelength scale for both curves, and trust the pattern across runs more than any single reading.

Steps

1. On each continuous curve, find where it cuts off sharply on the short-wavelength side; that meeting point with the axis is the shortest wavelength λ_{min}. Read it for the strong-voltage curve and for the weak-voltage curve.
2. Beside each reading record the accelerating voltage V printed on that curve (50 kV or 25 kV).
3. For each run work out the product $V \times \lambda_{min}$ and enter it in the last column.
4. Compare the products down the last column to judge whether $V \times \lambda_{min}$ stays the same as the voltage changes.

Run	Accelerating voltage V (kV)	Shortest wavelength λ_{min} ($\times10^{-11}$ m)	$V \times \lambda_{min}$ (kV·10^{-11} m)
strong-voltage curve			
weak-voltage curve			

Analyze what you collected:

(a) As the accelerating voltage V rises, does the shortest wavelength λ_{min} get longer or shorter? Now look down the last column — what do you notice about the product $V \times \lambda_{min}$?

(b) If $V \times \lambda_{min}$ is constant, then λ_{min} = (constant)/V. Say what that constant should equal in terms of h, c, and e, and use it to predict λ_{min} for a voltage you did not read.

✎ Write your explanation in two lines before you read the next page.

From observation to model

Continuous and characteristic X-rays

Two quite different mechanisms run inside the same tube, and the spectrum separates them by shape. An electron that is merely deflected as it plows into the target radiates away part of its energy; how much varies from electron to electron, so this contribution is smeared into a broad, smooth hump. An electron that instead knocks an inner electron clean out of a target atom starts something else: an outer electron drops into the vacancy and radiates the fixed energy gap between two levels of that atom. A fixed energy means a fixed wavelength, so this contribution appears as narrow spikes whose positions are a signature of the element used — change tungsten for copper and the spikes jump, while the hump keeps its shape. Only the deflection route has a ceiling: an electron arrives carrying eV of energy and can hand over, at most, all of it to a single photon, so the shortest wavelength obeys

$$eV = \frac{hc}{\lambda_0}$$

Raise the voltage and that edge slides to shorter wavelength — which is exactly the constant product measured in the last column of the Explore table.

Energy and momentum of a photon

Both photon relations are already in hand from Lesson 5-1 together with the wave equation $c = \nu\lambda$; the work here is only to pick the convenient form. Frequency is the natural variable when a packet is being absorbed by one electron, but X-ray apparatus is calibrated in wavelength, so eliminating ν between the two relations lets energies be read straight off a wavelength axis — the same photon described two ways, never two different photons. The momentum relation adds genuinely new information: a packet delivers not only energy but push. Momentum is a vector where energy is not, so an answer for p is unfinished until its direction is given, and that direction is simply the way the photon is going. This is why an absorbed beam presses on whatever stops it, and why the one constant h fixes both the energy scale and the momentum scale of light.

The electron volt

Units are chosen to fit the sizes actually being handled. Written in joules, every energy in this chapter sits near 10^{-19}, a range in which each calculation drags a power of ten about and no two numbers can be compared at a glance. Pushing one electron through one volt supplies a natural yardstick of precisely that size, so quoting energies as a multiple of it removes the exponent altogether: a tube run at thirty thousand volts gives each electron 30 keV, a figure read straight off the voltage dial with no conversion at all. The price is that this yardstick is not an SI unit — before it enters $E = hc/\lambda$, or any other formula, it has to be turned back into joules by multiplying by the elementary charge. Nuclear and particle physics work several powers of ten higher again, which is why their everyday unit carries the prefix mega.

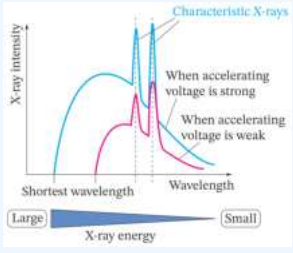
★ **Key fact**

Scale — the X-rays in this lesson have wavelengths near 10^{-11} m, some ten thousand times shorter than green light and comparable with the spacing of atoms in a crystal. That coincidence is what makes X-ray diffraction the standard way of measuring crystal structure, and the same photon energies are large enough to break molecular bonds — hence the lead apron.

Point

! The absolute value of the charge held by one electron is called the **elementary charge**, and is represented by the symbol e.

X-rays are electromagnetic waves with wavelengths shorter than those of ultraviolet rays.



The relationship between the intensity and wavelength of X-rays obtained by striking a metal with electrons accelerated by a high voltage is schematically shown in the figure above. The smooth part of the curve is called continuous X-rays, and the parts where the intensity is prominent are called **characteristic X-rays**. The wavelengths of characteristic X-rays are determined by the type of metal the electrons strike. (The smaller the wavelength, the larger the energy.) When all the energy of an electron is converted into the energy of one X-ray photon, the energy of the X-ray photon becomes **maximum**, and the wavelength becomes **shortest**.

The energy E [J] of a single photon is proportional to the frequency of light ν [Hz].

$E = h\nu$ (h : Planck's constant) — substituting $\nu = c/\lambda$, we get $E = hc/\lambda$.

The momentum p [kg·m/s] of a single photon is inversely proportional to the wavelength of light λ [m].

$p = \dfrac{h}{\lambda}$ — substituting $\lambda = c/\nu$, we get $p = h\nu/c$.

The direction of momentum is the same as the direction the photon travels.

Since the energy of one electron or one photon is extremely small, the **electron volt** (symbol **eV**) is sometimes used as a unit of energy besides the joule (symbol J). 1 eV is equal to the energy an electron with a charge of $-e$ [C] gains when accelerated by a voltage of 1 V.

1 eV = e [J]

Also, 10^6 eV is called a **mega-electron volt** (symbol **MeV**).

Equation box

Photon energy	$E = h\nu = \dfrac{hc}{\lambda}$ (h = Planck's constant; unit: J)
Photon momentum	$p = \dfrac{h}{\lambda} = \dfrac{h\nu}{c}$ (unit: kg·m/s, directed along the photon's travel)
Shortest wavelength	$eV = \dfrac{hc}{\lambda_0} \Longrightarrow \lambda_0 = \dfrac{hc}{eV}$ (all electron energy ⟶ one photon)
Electron volt	1 eV = e J (energy gained by charge $-e$ across 1 V); 1 MeV = 10^6 eV

★ Term

Where the model stops — the cut-off relation assumes one electron gives everything to one photon, which almost never happens: most electrons make several soft photons or simply heat the target, so an X-ray tube is a poor converter and its anode must be cooled. The relation therefore predicts an edge, not the height or shape of the spectrum, and it says nothing at all about where the characteristic lines fall — those come from the target atom's internal levels, a subject the photon picture alone cannot supply.

★ Note

Careful — a question about "the wavelength of the X-rays" has two possible targets on the same graph. The cut-off belongs to the whole spectrum and slides when the tube voltage is altered; the wavelengths of the sharp peaks belong to the target metal and do not move at all. Reading a peak when the cut-off was wanted is the commonest error here. Check too that the voltage is in volts and not kilovolts before it goes into any calculation.

🛠 Worked Example

Situation	Determine the following to 2 significant figures. Take Planck's constant as 6.6×10^{-34} J·s, the speed of light as 3.0×10^{8} m/s, and the elementary charge as 1.6×10^{-19} C. (a) Electrons accelerated by a voltage of 2.2×10^{4} V strike a metal. Determine the shortest wavelength, in m, of the generated X-rays. (b) An X-ray photon of wavelength 1.5×10^{-10} m travels in the positive direction of the x-axis. Determine the direction and magnitude of its momentum, in kg·m/s.
Given and required	(a) $V = 2.2\times10^{4}$ V, $h = 6.6\times10^{-34}$ J·s, $c = 3.0\times10^{8}$ m/s, $e = 1.6\times10^{-19}$ C; find the shortest wavelength λ_0. (b) $\lambda = 1.5\times10^{-10}$ m along $+x$; find the direction and magnitude of p.
Representation	The shortest wavelength occurs when all of one electron's energy is converted into the energy of one photon. An electron accelerated through V gains kinetic energy eV (from potential energy $U = qV$ with $q = e$), which equals the maximum photon energy hc/λ_0. For (b), a photon's momentum points along the direction it travels.
Strategy	Set the electron energy equal to the photon energy, $eV = hc/\lambda_0$, and solve for $\lambda_0 = hc/(eV)$. For the momentum, use $p = h/\lambda$ directly, and keep the direction of travel.
Annotated solution	**(a)** When all the electron's energy becomes one photon's energy, the wavelength is shortest. The electron's energy is eV [J], so with the shortest wavelength λ_0, since $eV = hc/\lambda_0$, $\lambda_0 = hc/(eV)$. $\lambda_0 = (6.6\times10^{-34})(3.0\times10^{8}) / [(1.6\times10^{-19})(2.2\times10^{4})] = (1.98\times10^{-25})/(3.52\times10^{-15}) = 5.625\times10^{-11} \approx$ **5.6×10^{-11} m**. **(b)** Letting the momentum be p [kg·m/s], $p = h/\lambda = (6.6\times10^{-34})/(1.5\times10^{-10}) = 4.4\times10^{-24}$, so **$4.4\times10^{-24}$ kg·m/s in the positive direction of the x-axis**.
Reasonableness check	(a) Substituting back, $hc/\lambda_0 = (1.98\times10^{-25})/(5.625\times10^{-11}) = 3.52\times10^{-15}$ J, and $eV = (1.6\times10^{-19})(2.2\times10^{4}) = 3.52\times10^{-15}$ J — the two energies match. (b) A shorter wavelength would give a larger momentum; 1.5×10^{-10} m is a typical X-ray wavelength and yields a small momentum of order 10^{-24} kg·m/s, as expected for a single photon.

🧐 **Explain it yourself**	In one sentence, explain why increasing the accelerating voltage V makes the shortest wavelength λ_0 smaller.

✍️ **Now you try:** An X-ray photon has a wavelength of 3.0×10^{-11} m. Find its energy using $E = hc/\lambda$.

🔆 Try

1. Determine the following to 2 significant figures. Take Planck's constant as 6.6×10^{-34} J·s, the speed of light as 3.0×10^{8} m/s, and the elementary charge as 1.6×10^{-19} C. Electrons accelerated by a voltage of 5.0×10^{4} V strike a metal. Determine the shortest wavelength, in m, of the generated X-rays.
2. An X-ray photon of wavelength 6.0×10^{-10} m travels in the negative direction of the x-axis. (i) Determine the energy of this X-ray photon, in J. (ii) Determine the direction and magnitude of its momentum, in kg·m/s.

🌐 In a new context

A patient's chest X-ray uses exactly this physics. The machine accelerates electrons through a chosen tube voltage; a higher voltage produces X-rays with a smaller shortest wavelength and therefore higher photon energy, which penetrate tissue more strongly. Radiographers pick the voltage to match the body part: a lower voltage for a hand, a higher one for a chest, so that enough photons pass through to form a clear image on the detector without using more energy than needed.

Figure 5-2-3 · During a chest X-ray, an overhead tube directs a cone-shaped beam from its small focal spot through the patient's chest onto a flat detector, while a radiographer works behind a protective screen; the monitor shows dense bone as bright white against darker, X-ray-transmitting lung fields.

➤ **Your turn:** A tube is set to an accelerating voltage of 4.0×10^{4} V. Using $\lambda_0 = hc/(eV)$, estimate the shortest wavelength of its X-rays.

🧪 Nature of Science

From the original — how X-rays see through the body. X-rays are electromagnetic waves with very short wavelengths and a strong ability to pass through matter (penetrating power). However, they do not easily pass through high-density substances that contain a lot of calcium, such as bones, so only the bone parts appear as white shadows on photographic film or a sensor, making it possible to see inside the body.

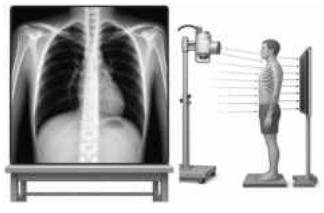


Figure 5-2-4 · X-rays from the source pass through soft tissue but are stopped by dense bone, so the bones print as bright white shadows on the detector.

Science advances when a quantity that cannot be seen — here, the photon energy carried by an X-ray — is tied by a single equation, $E = hc/\lambda$, to something measurable, the wavelength. The same photon model that sets the shortest wavelength in a tube also predicts how deeply the radiation penetrates tissue, letting engineers design a safe, useful medical image from first principles.

Röntgen came upon these rays by accident in November 1895 while working with discharge tubes, and the discovery brought the first Nobel Prize in Physics in 1901. Duane and Hunt later tied the short-wavelength edge to the tube voltage — the very relation used here. An accident becomes a discovery only when someone follows it up carefully.

Your turn: Suggest one measurement on the intensity–wavelength spectrum that would let you identify which metal was used as the target.

Lesson question, answered — In a tube, not in nature. Electrons are pulled through a large potential difference and slammed into a metal target, where their kinetic energy returns as radiation by two routes: deflection, which yields the broad smooth hump, and the refilling of inner atomic vacancies, which yields the sharp lines belonging to that particular metal. The voltage decides how much energy each electron brings, and so decides how far to the short-

The scene we opened with.

wavelength side the hump can reach; the choice of target decides where the sharp lines fall. Because the resulting packets are far more energetic than those of light, they slip through soft tissue and are halted by bone — which is what draws the picture.

📋 Exercise

1. Determine the following to 2 significant figures. Take Planck's constant as 6.6×10^{-34} J·s, the speed of light as 3.0×10^{8} m/s, and the elementary charge as 1.6×10^{-19} C. Electrons accelerated by a voltage of 3.0×10^{4} V strike a metal. Determine the shortest wavelength, in m, of the generated X-rays.
2. The intensity–wavelength spectrum of the X-rays generated by striking a metal with accelerated electrons is shown in the figure on the right; its sharp short-wavelength cut-off lies at 3.0×10^{-11} m. Determine the accelerating voltage *V*, in V, used to generate these X-rays.

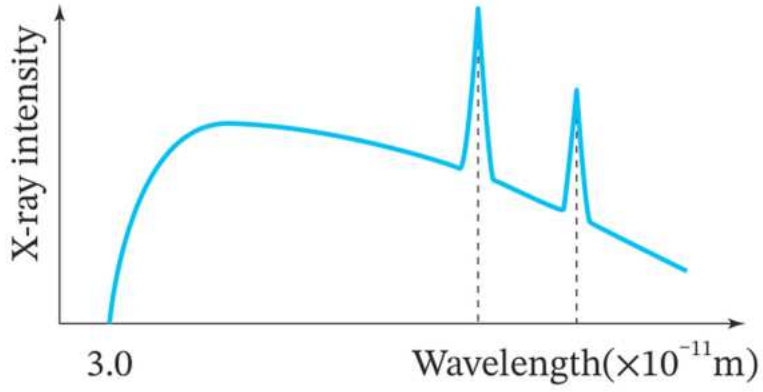


Figure 5-2-5 · Exercise 2: the intensity–wavelength spectrum of the X-rays, with a sharp cut-off at 3.0×10^{-11} m and two characteristic peaks.

3. Electrons are accelerated by a voltage of 1.0×10^{5} V and strike a metal. (i) Determine the energy of the accelerated electrons, in J. (ii) Determine the shortest wavelength, in m, of the generated X-rays. (iii) Determine the momentum, in kg·m/s, of the X-ray photons of that shortest wavelength.

5-3 Matter Waves

❓ **Lesson question:** Just as light behaves as both a particle and a wave, can a particle such as an electron also behave as a wave?

You will learn to

1. Say what it means for something with mass to have a wavelength at all, and name that wave in the particular case of an electron.
2. Get a number out of it: turn a given mass and speed into a wavelength, and judge the answer against the size of an atom.
3. Argue from that number why an electron microscope resolves detail no light microscope can reach.

🔍 The phenomenon — predict before you continue

Send a narrow beam of fast electrons through a thin sheet of graphite and let it land on a screen. You might expect a single bright spot where the particles pile up. Instead the screen shows a set of concentric bright rings, exactly the kind of pattern that light makes when it passes through a fine grating. The stream of "particles" has spread and interfered like a wave.

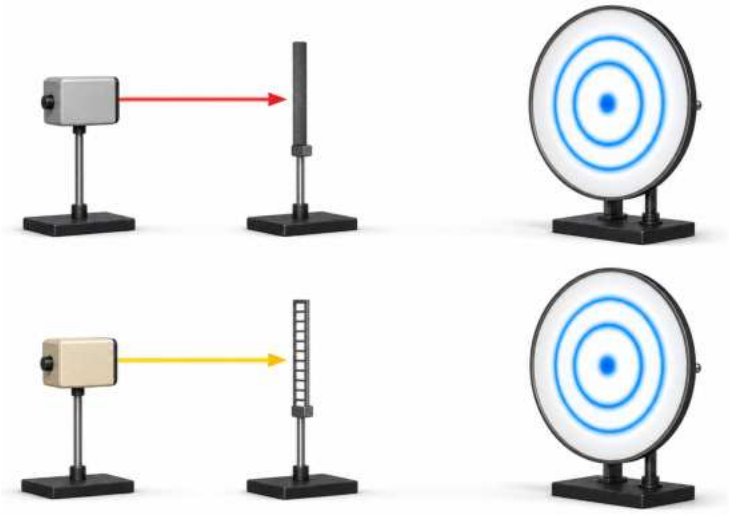


Figure 5-3-1 · An electron beam through a graphite film makes the same concentric diffraction rings that light makes through a grating — the electron beam travels as a wave.

Predict

(a) If the electrons in the beam are made to move faster, so their momentum p is larger, do you expect the rings to spread wider apart or to crowd closer together?

(b) Would a beam of much heavier particles moving at the same speed show a wider or a narrower ring pattern?

★ **Key idea**

Matter wave — the wave–particle trade runs in both directions: what was thought to be purely a wave turns out to carry momentum, and what was thought to be purely a particle turns out to have a wavelength.

🔍 Explore — Size the electron's wavelength from its rings

What you'll do: send an electron beam through a graphite film at several accelerating voltages and measure how the diffraction-ring size changes as the electrons' momentum grows.

Time: 15 min

You will need:

- an electron-diffraction tube
- a high-voltage dc supply
- a voltmeter
- a ruler

diffraction tube

High-voltage power supply

voltmeter

ruler

Steps

1. Set the high-voltage power supply to a known accelerating voltage V and read it on the voltmeter. The faster the electrons, the larger their momentum $p = mv$.
2. With the ruler, measure the diameter D of one chosen bright ring on the screen.
3. Raise V in steps and, for the same ring, record V and its diameter D each time.
4. For each run work out $D \times \sqrt{V}$ and enter it in the last column.

Run	Voltage V (kV)	Ring diameter D (cm)	$D \times \sqrt{V}$ (cm·$\sqrt{\text{kV}}$)
1			
2			
3			

Analyze what you collected:

(a) As the accelerating voltage V rises, what happens to the ring diameter D? Look down the last column — what do you notice about the product $D \times \sqrt{V}$?

(b) A ring's diameter D grows with the wavelength λ, and a larger V gives a larger momentum p. From the near-constant $D \times \sqrt{V}$, argue how λ depends on p, and predict the rings a much slower electron beam would make.

✎ Write your explanation in two lines before you read the next page.

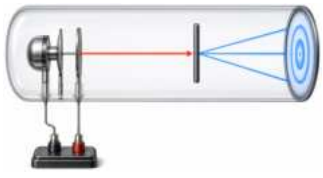


Figure 5-3-2 · An electron-diffraction tube: the gun fires electrons through the accelerating voltage V onto a graphite target; the ring radius on the screen is measured.

★ **Working note**

Darken the room, and each time measure the *same* bright ring across its center from inner edge to inner edge; trust the pattern across runs more than any single reading.

From observation to model

A moving particle carries a wave

The argument that produced this idea was one of symmetry. Light had spent a century as a wave and then, in the photoelectric effect and in X-ray scattering, was forced to behave as something carrying momentum

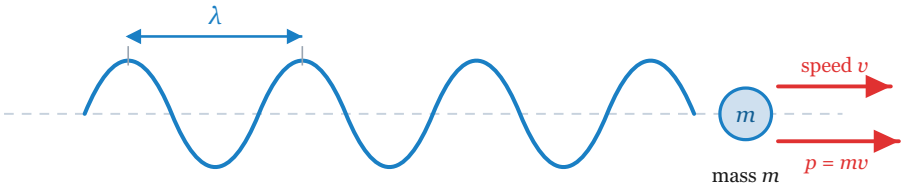
$$p = \frac{h}{\lambda}$$

De Broglie asked why nature would use that relation in one direction only, and read it backwards: given something of known momentum, solve the same equation for λ. Nothing in that step is a proof — it is a guess, but a guess with a sharp and testable

consequence. If a beam of electrons really carries a wavelength of atomic size, then firing it at a crystal, whose atomic planes happen to be spaced at about that same size, must produce the ring pattern a diffraction grating produces with light. The rings in Figure 5-3-1 are that test, passed.

Wavelength set by momentum

Read the formula as a rule about its denominator. Everything that makes something harder to deflect — more mass, more speed — sits underneath Planck's constant, so every such increase shrinks the answer; and since the constant on top is itself of order 10^{-34}, even a very small denominator leaves a very small result. That one fact decides where the effect can be seen at all. To get a wavelength big enough to diffract, the momentum has to be minute, which in practice means something as light as an electron pushed by no more than a modest voltage. Raising the accelerating voltage in the Explore does the reverse: it grows the momentum, shrinks the wavelength, and pulls the rings inward — which is precisely why the product measured in the last column stayed put.



$$\lambda = h/p = h/(mv)$$

$h \approx 6.63\times10^{-34}$ J·s

Figure 5-3-3 · The two descriptions drawn together: a lump of mass m traveling at speed v, and the wave that runs along the same path, its crest spacing fixed by the momentum.

Point

Particles have wave properties. This is called a **matter wave**. In particular, when the particle is an electron, the wave is called an **electron wave**. Let the magnitude of the particle's momentum be p [kg·m/s], the mass of the particle be m [kg], and the speed be v [m/s], the wavelength λ [m] of the matter wave is expressed as $\lambda = \frac{h}{p} = \frac{h}{mv}$ (h : Planck's constant).

Equation box

Matter wavelength	$\lambda = \frac{h}{p} = \frac{h}{mv}$ (unit: m; h = Planck's constant [J·s])
Momentum	$p = mv$ (unit: kg·m/s)
Constants	$h \approx 6.63\times10^{-34}$ J·s electron mass $m = 9.1\times10^{-31}$ kg

★ Key fact

Scale — the electron of the Worked Example comes out at about 7×10^{-11} m, roughly the width of an atom; a thrown tennis ball comes out near 10^{-34} m, far below any length that can be measured or that anything could diffract around. Both answers come from the same formula. Wave behavior is not rare or exotic — it is universal, and for anything heavier than an atom it is utterly invisible.

★ Note

Careful — work in SI units throughout: a mass slipped in as grams, or a speed as km/s, moves the answer by whole powers of ten, and at this scale the power of ten *is* the answer. Divide by the momentum, never by the speed alone. And when a question supplies an accelerating voltage instead of a speed, find the momentum first from the energy the charge gains, then divide.

🛠 Worked Example

Situation	Determine the wavelength of an electron wave with a speed of 1.0×10^{7} m/s, in m. Assume Planck's constant is 6.6×10^{-34} J·s and the mass of an electron is 9.1×10^{-31} kg.
Given and required	$v = 1.0\times10^{7}$ m/s, $m = 9.1\times10^{-31}$ kg, $h = 6.6\times10^{-34}$ J·s; find the wavelength λ.
Representation	The electron carries a matter wave whose wavelength is fixed by its momentum $p = mv$.
Strategy	First find the momentum $p = mv$ from the mass and speed of the particle, then substitute it into the matter-wave formula $\lambda = h/p$ to calculate the wavelength.
Annotated solution	Momentum: $p = mv = (9.1\times10^{-31})\times(1.0\times10^{7}) = 9.1\times10^{-24}$ kg·m/s. Wavelength: $\lambda = h/p = 6.6\times10^{-34}$ / $[(9.1\times10^{-31})\times(1.0\times10^{7})] = 6.6\times10^{-34}$ / 9.1×10^{-24} $\lambda \approx$ **7.3×10^{-11} m**.
Reasonableness check	The wavelength is about 10^{-10} m, roughly the size of an atom and far shorter than visible light (about 5×10^{-7} m). A wavelength this small is exactly what lets an electron beam resolve atomic-scale detail.
😊 Explain it yourself	In one sentence, say why speeding the electron up makes its matter wavelength shorter.

✍️ **Now you try:** Determine the wavelength of the matter wave of an object of mass 0.10 kg moving at a speed of 30 m/s, in m. Assume Planck's constant is 6.6×10^{-34} J·s.

🔆 Try

1. Determine the wavelength of an electron wave with a speed of 2.0×10^{6} m/s, in m. Give the answer to 2 significant figures, assuming Planck's constant is 6.6×10^{-34} J·s and the mass of an electron is 9.1×10^{-31} kg.

🌐 In a new context

An electron microscope puts the matter wave to work. Because an electron wave can be given a wavelength thousands of times shorter than that of visible light, an instrument that focuses electron waves instead of light rays can clearly image extremely small objects — viruses and fine surface structures — that an ordinary optical microscope can never resolve. The shorter the wavelength, the finer the detail that can be seen.

➤ **Your turn:** Compare the electron wave from the Worked Example ($\lambda \approx 7.3\times10^{-11}$ m) with green light ($\lambda \approx 5\times10^{-7}$ m). About how many times shorter is the electron wavelength, and why does that let the electron microscope see finer detail?

Nature of Science

From the original — Turning a strange idea into a working tool. The claim that a particle is also a wave sounds like a paradox, yet it was accepted because it was tested: electron beams were found to diffract, producing wave-like ring patterns, and their measured wavelengths matched $\lambda = h/p$. Once the relation survived that test, it was not left as a curiosity — it became the operating principle of the electron microscope, an instrument that only works if electrons truly behave as short-wavelength waves. A model earns trust when its surprising prediction is confirmed by measurement and then does reliable work.

De Broglie proposed this in his thesis of 1924 with no experiment behind it, on symmetry alone; Davisson and Germer, and G. P. Thomson, diffracted electrons in 1927 and turned the guess into physics. A bold hypothesis is welcome in science, but it earns its place only when a measurement agrees with it.

Your turn: Suggest one observation an experimenter could make with an electron-diffraction tube to test the prediction that a faster electron beam has a shorter wavelength.

Lesson question, answered — Yes, and the evidence is a photograph. Electrons fired at a graphite film land in concentric rings, which is what only something spreading and interfering can do; a stream of hard little balls would make one spot. The ring sizes agree with the wavelength predicted from the electrons' momentum, and they shrink when the accelerating voltage is raised, exactly as predicted. The reason the effect stays hidden in daily life is arithmetic, not principle: divide Planck's constant by the momentum of anything you can hold and the result is smaller than any object it could bend around. The same short wavelength that makes the effect invisible for a tennis ball is what makes the electron microscope possible.

Exercise

1. Consider an electron of mass m and charge $-e$ accelerated from rest through a voltage V. Assume Planck's constant is h.
   (i) Determine the momentum of the accelerated electron.
   (ii) Determine the wavelength of the electron wave.

Figure 5-3-4 · A transmission electron microscope sends a fine electron beam straight down an evacuated column, focused by ring-shaped magnetic lens coils onto a tiny specimen, producing a highly magnified grayscale image of a virus resolved far beyond the reach of any light microscope.

★ **Term**

Electron microscope — an instrument that uses electron waves, whose wavelength is much shorter than that of light, to image extremely small objects clearly.

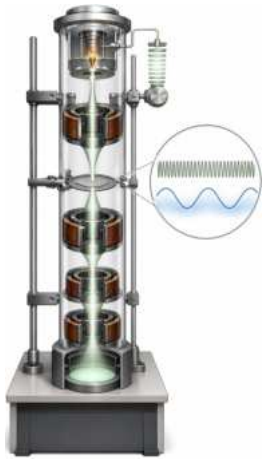
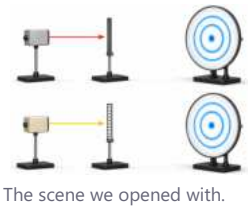


Figure 5-3-5 · An electron-microscope column focuses the electron beam with magnetic lenses onto the specimen; the inset compares the very short electron wavelength (green) with the far longer wavelength of light (blue) — that is why the electron beam resolves fine detail light cannot.



The scene we opened with.

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